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LOGIC

A Complete Introduction

Siu-Fan Lee

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Meet the author

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An analytic philosopher by training, Siu-Fan also appreciates diversities such as ideas in Continental Philosophy and Chinese Philosophy. She is active in seeking a comprehensive understanding and dialogue between different perspectives and traditions.

Introduction

Human beings are interested not only in what things are, but also in how things *could be*. To achieve the latter, we need to understand how things are connected and why, so that we can play around with possibilities and derive things we do not know yet from what is known already. Hence, for instance, we deduce that Socrates is mortal because all men are mortal and Socrates is a man. We see black clouds looming and know that rain will come soon because we reason that if there are black clouds, then it rains, and now there are really black clouds. Similarly, we observe most swans are white, so we infer that all swans are white (even though in fact they are not). All these examples involve reasoning or inference. Ever since humans have sought to answer 'why' we believe this and that, we have been constantly engaged in the truth of individual statements as well as the connections between them.

Logic studies reasoning; it is the study of the methods and principles to distinguish good from bad reasoning. Very often the relationship between individual statements depends on the form or structure of the arguments, rather than their actual content. Logic focuses on studying the forms or structures of arguments. So logic is abstract. However, this feature also gives logic a universal and general character. Exactly because content does not matter, logic is widely applicable in all intellectual pursuits regardless of their subject.

This book introduces three basic logical systems: categorical logic, propositional logic and predicate logic. It identifies in each system an essential method to test arguments: the Venn diagram method for categorical logic, the truth-table method for propositional logic, and natural deduction for predicate logic. There are plenty of illustrations and exercises to enhance your mastery of these skills. Each system involves an insight into analysing ordinary language, too, so logic can be related to our everyday use of language. Categorical logic analyses the subject-predicate structure within a sentence. Propositional logic focuses on the connections between simple sentences.

Predicate logic combines insights from both, and provides a very powerful tool to analyse complex thoughts and relations. These systems and methods are chosen for their intuitive appeal and wide applicability to everyday argumentation. Advanced readers, however, may want to go deeper and get to know other complicated systems.

Apart from formal knowledge, this book also discusses the nature of meaning. It introduces some informal fallacies about meaning, the use of language and the relationship between premises and conclusion. This is useful for identifying bad reasoning in our daily lives. This book is therefore positioned between critical thinking and logic – more formal than a standard critical thinking course, but less technical than a pure logic curriculum.

This book aims not only to help you to master the methods in elementary logic but also to understand the ideas behind it. By the time you have finished reading the book, I hope that you will understand the way in which logicians think and start to think like them! Of course, all systems have limitations, and it's important that you understand what these are in order to apply the logic skills appropriately in your daily life.

In the book, I adopt a dialogue approach and invite you to follow me in my inquiry; there will be opportunities to practise what you have learned with exercises, and to consolidate what you have learned with reflection questions. You do not need to have any prior background in logic or mathematics to understand this book. Suggested answers for exercises are provided at the back of the book. Key technical terms are in bold in the text and explained again in the Glossary at the back of the book.

1

What is logic?

In this chapter you will learn about:

- ***what logic is***
- ***what an argument is***
- ***deduction and induction***
- ***truth and validity***
- ***the study of logic***

> *'It used to be said that God could create anything except what would be contrary to the laws of logic – The truth is that we could not say what an "illogical" world would look like.'*
>
> Ludwig Wittgenstein (1921), *Tractatus Logico-Philosphicus*, 3.031

1.1 What is logic?

Logic is the study of the methods and principles used to distinguish between good and bad reasoning. It is a **normative** discipline, in the sense that it does not survey and describe *how* we actually reason (which is the job of the psychologist) but what we *should* do in reasoning.

Reasoning concerns arguments. In the first part of this chapter we explain what an argument is made up of and how to identify an argument. Then we introduce the distinction between two basic types of reasoning: deduction and induction. Since we often talk of an argument being true or valid (indeed it is technically wrong to describe an argument as true, we shall explain later), we discuss the relation between truth and validity in Section 1.4. Finally, we look at certain misconceptions about logic and reasoning to allay any unnecessary fears about logic.

1.2 What is an argument?

STRUCTURE

An **argument** is a structure that comprises a **conclusion,** namely, a proposition that one wants to uphold, and some **premises,** as reasons to support the belief in the conclusion. Logic is about whether and how a conclusion follows from the premises. What it means for a conclusion (a certain proposition B) to 'follow' from a premise (another proposition A) is sometimes cashed out as whether whenever the premise (A) is true, the conclusion (B) is also true. Arguments come in many forms in real life; sometimes they are easy to spot but sometimes they are covert. We thus need first of all some training to heighten our awareness of and ability to identify arguments.

An argument contains one and only one conclusion. In fact, an argument is individuated by its conclusion. If we want to make more than one conclusion, then there must be more than one argument. An argument can be represented in the following standard form: the premises are listed on the top followed by the conclusion at the end and each sentence is numbered for easy reference. We also use a line to separate the premises and conclusion.

Premise 1

Premise 2

...______

Conclusion

A conclusion is supposed to be supported by the premises. There may be many premises in an argument. Indeed, there is no limit to the number of premises. However, is there a minimum number of premises in an argument? If so, what is it? One, two, or three? Surprisingly, formally, the answer to the minimum number of premises required is zero! This is surprising because premises are reasons to support a conclusion and allowing zero premises would mean that the conclusion is supported by nothing, and that seems to directly contradict the goal of reasoning, namely, to accept an argument only if one can provide a reason to believe in it. These are legitimate points. Yet when logicians allow an argument to have no premises, it merely means that *nothing goes against* the conclusion, rather than that the conclusion is supported by no reason. We regard such a conclusion as self-evident. The following are some examples of self-evident truths.

Example (1): Everything is identical to itself.

Example (2): Something is the case or it is not the case.

Example (3): It cannot be true that something is both the case and not the case at the same time.

Example (4): An object cannot be red and green all over at the same time.

Self-evident truths are obvious truths that do not need *anything* to support them. Equally, it can also be said that *everything* supports them because nothing counts as a reason to reject them, and they do not contradict with anything. Later we introduce the idea that an argument is valid if it preserves truth from the premises to the conclusion. Given this definition, because a conclusion containing a self-evident truth is always true, any statement serving as its premise, if true, will lead to the truth of the conclusion. Thus, every argument supporting a self-evident truth is a valid argument!

Examples (1)–(3) are so general and intuitive that their general forms are accepted as **laws of logic**. The three logical laws are named respectively:

> Example (1) The Law of Identity
>
> Example (2) The Law of Excluded Middle
>
> Example (3) The Law of Contradiction
>
> Example (4) is an example of a necessary truth in metaphysics.

Some philosophers even claim that contingent truths can be self-evident truths as long as they are very intuitive and hardly anything counts to refute them. For example, first person perceptual truths, such as (5), are self-evident.

> Example (5): This is my hand (while I hold it up and look at it closely in normal perceptual circumstances) and this is another (waving it).

Self-evident truths are special cases. Most arguments are not like that but contain a conclusion and at least one premise.

More examples of an argument are given below when we introduce the techniques in identifying an argument.

Key idea: Self-evidence

Is self-evidence just the same as having nothing to go against it? How do you take it?

The question of self-evidence is more complicated than it looks. Self-evident truths are intuitive. It is difficult to argue against widespread intuition, especially when many other judgements are based on it. Consider what I am to do if I really doubt that these hands are mine? I will also need to doubt many other things that we so take for granted in our lives. Yet we do know that sometimes what we believe is true may turn out to be false; for example, I could mistake a robot dog seen in the distance for a real dog. How to distinguish various types of intuition and more importantly, how to justify those very general and foundational ones? These are not easy questions!

A philosopher Ludwig Wittgenstein once described the situation as: 'If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what I do."' (Ludwig Wittgenstein (1958), *Philosophical Investigations*, section 217)

Moreover, should we treat self-evidence the same as having nothing to go against it? When we do so, don't we already assume that a proposition is either true or false, and then nothing is both true or false? Yet these laws of logic (the Law of Excluded Middle and the Law of Contradiction respectively) are exactly some of the self-evident truths that we claim to exist. So we would be defining self-evident truth using some self-evident truths. Isn't that begging the question?

For our very question is that although we do not usually challenge self-evident truths, it seems hard to explain why self-evident truths cannot be challenged. Indeed, some laws of logic are challenged under different systems of logic. The three logical laws stated above are the laws for classical logic. There are non-classical logic systems that try to answer these questions in a different way. For instance, paraconsistent logic does not accept the Law of Contradiction – some paraconsistent logicians claim that there are true contradictions in reality, such as a group of people holding opposing moral views; some claim that all contradictions are trivially true.

TECHNIQUES IN IDENTIFYING AN ARGUMENT

Since all arguments have one and only one conclusion, to identify an argument we should *identify the conclusion first* and then trace back to the premises supporting it. Consider the following arguments. Can you identify the conclusion and the premises?

> Example (6): It has been raining for a month now. So, it is likely to rain again tomorrow.
>
> Example (7): If God exists, then there would not be any suffering. There is suffering. Therefore, God does not exist.
>
> Example (8): Mr Smith must be either a monster or a strict teacher because only a monster or a strict teacher would beat children, and he beats them.

For (6), the conclusion is the second sentence and the premise is the first sentence. If we rewrite it into the standard form, we get the following:

> <u>(Premise) It has been raining for a month now.</u>
>
> (Conclusion) So, it is likely to rain again tomorrow.

(7) has one conclusion and two premises. The conclusion is the last sentence.

> (Premise 1) If God exists, then there would not be any suffering.
>
> <u>(Premise 2) There is suffering.</u>
>
> (Conclusion) Therefore, God does not exist.

(8) also has one conclusion and two premises; however, the conclusion is stated at the front, followed by the premises. Moreover, although the whole inference is expressed in one sentence, we can identify individual parts as premises and conclusion.

> (Premise 1) Only a monster or a strict teacher would beat children.
>
> <u>(Premise 2) Mr Smith beats children.</u>

(Conclusion) Mr Smith must be either a monster or a strict teacher.

Notice the myriad ways in which ordinary language may present an argument. The conclusion can be stated at the end (as in (6) and (7)) or at the front (as in (8)). An argument can have a simple sentence as a conclusion (as in (6) and (7)) or a complex one which contains more than one set of subject–predicate relations (as in (8)). Furthermore, an argument can contain one (6) or more than one premises (as in (7) and (8)).

While the structure of an argument varies a lot, there are often words typically indicating which sentence is a premise and which is the conclusion. These words are called **premise-indicators** and **conclusion-indicators** respectively. The following are some common examples.

- **Premise-indicators:** because, since, for, as, follows from, as shown by, inasmuch as, as indicated by, the reason is that, for the reason that, may be inferred/derived/deduced from, in view of the fact that...
- **Conclusion-indicators:** therefore, so, hence, thus, in consequence, consequently, accordingly, as a result, for this reason, it proves that, it follows that, we may infer, which allow us to infer that, which shows/means/entails/implies that, which points to the conclusion that...

It is helpful to look for these indicators in a passage to help you identify an argument.

MISSING PREMISES OR CONCLUSION: ELLIPTICAL ARGUMENTS

Sometimes an argument may have a missing premise, or premises, or even a missing conclusion. These arguments are called elliptical arguments. In such cases, one has to fill in the missing parts in order to reveal the full reasoning behind them. Doing so often involves contextual interpretation and as such controversy may arise as to whether the interpretation is correct. Below is an example.

Example (9): Why are you still here? All students should report to the examination hall 15 minutes before the examination begins.

It represents the following argument:

> (Premise 1) All students should report to the examination hall 15 minutes before the examination begins.
>
> (Premise 2, missing) You are a student going to sit in an examination now.
>
> (Premise 3, missing) You are here.
>
> (Premise 4, missing) Here is not the examination hall.
>
> (Conclusion, implied) You should not be here.

Example (9) is also an example of a rhetorical question. Sometimes a question is asked or an exclamation is made, yet the point of the utterance is not really to enquire or to express emotion. Rather, an implicit proposition is presented, though in the form of a different mood of speech. The question 'why are you still here?' in (9) does not just ask for an answer but really implies that the speaker assumes the audience should not be here. In clarifying an argument, we often need to grasp the implied or hidden meaning from relevant context, make it explicit and state it in the form of a proposition or an assertion instead.

CHAIN ARGUMENTS OR SUB-ARGUMENTS

Sometimes there may be arguments within a bigger argument. We call the smaller arguments **sub-arguments**. Each sub-argument generates a conclusion and the conclusion is used again as a premise to another argument, such that eventually a different, usually more general, conclusion is reached. Because the conclusion of a sub-argument is linked to other arguments, sometimes the whole argument can also be called a **chain argument**, though indeed the term technically refers to a particular form of argument called hypothetical syllogism.

Below is an example of hypothetical syllogism.

> Example (10): Without the community, individuals won't flourish. Yet communities won't exist without a state. Hence, without a state, individuals won't flourish.

This argument can be reconstructed as follows.

(Premise 1) If there is no state, there is no community.

(Premise 2) If there is no community, individuals won't flourish.

(Conclusion) Therefore, if there is no state, individuals won't flourish.

It is a chain in the sense that one element leads to another: the absence of a state leads to the absence of a community, which then leads to the absence of opportunities for individuals to flourish. If we represent the different elements with a symbol (A, B, C, etc.), the elements are linked as in a chain. For instance, suppose we let A stand for the absence of a state, B the absence of a community, C the absence of individuals flourishing. The argument form can be abstracted as follows:

$A \rightarrow B$. $B \rightarrow C$. Therefore, $A \rightarrow C$.

Below are examples of arguments having sub-arguments.

Example (11): Every person lives only once. You are a person. If you live only once, you should treasure every moment of your life. Therefore, you should treasure every moment of your life.

The argument has the following form:

(Premise 1) Every person lives only once.

(Premise 2) You are a person.

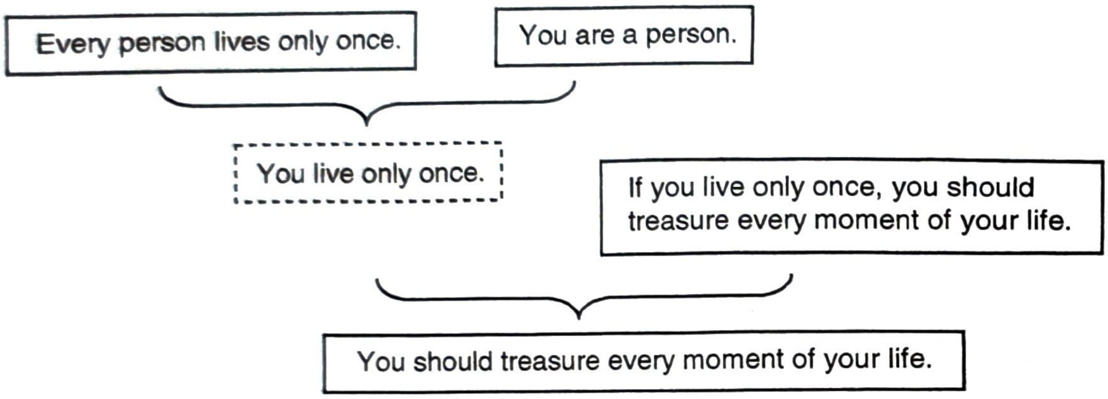
(Conclusion, implicit) You live only once.

(Premise 3) If you live only once, you should treasure every moment of your life.

(Premise 4, implicit) You live only once. (The implicit conclusion of premise 1 and 2)

(Conclusion) Therefore, you should treasure every moment of your life.

So premises 1 and 2 already form an argument to infer the conclusion that you live only once. Using that conclusion as a premise, the reasoner goes on to argue further that you should treasure every moment of your life. The latter is the ultimate conclusion of the whole argument. This whole argument has a sub-argument, formed by premises 1 and 2, to generate the required result. We can also use a diagram to show the structure of the argument. This clearly indicates the role of the sub-argument in the inference.



Example (12): Moriarty is likely to be the culprit. This is because only a criminal mastermind can plan and execute such an elaborate crime and he is proven to be a criminal mastermind. Moreover, he was actually present at the crime scene.

(Premise 1) Only a criminal mastermind can plan and execute such an elaborate crime.

(Premise 2) Moriarty is a criminal mastermind.

(Conclusion, implicit) Moriarty can plan and execute such an elaborate crime.

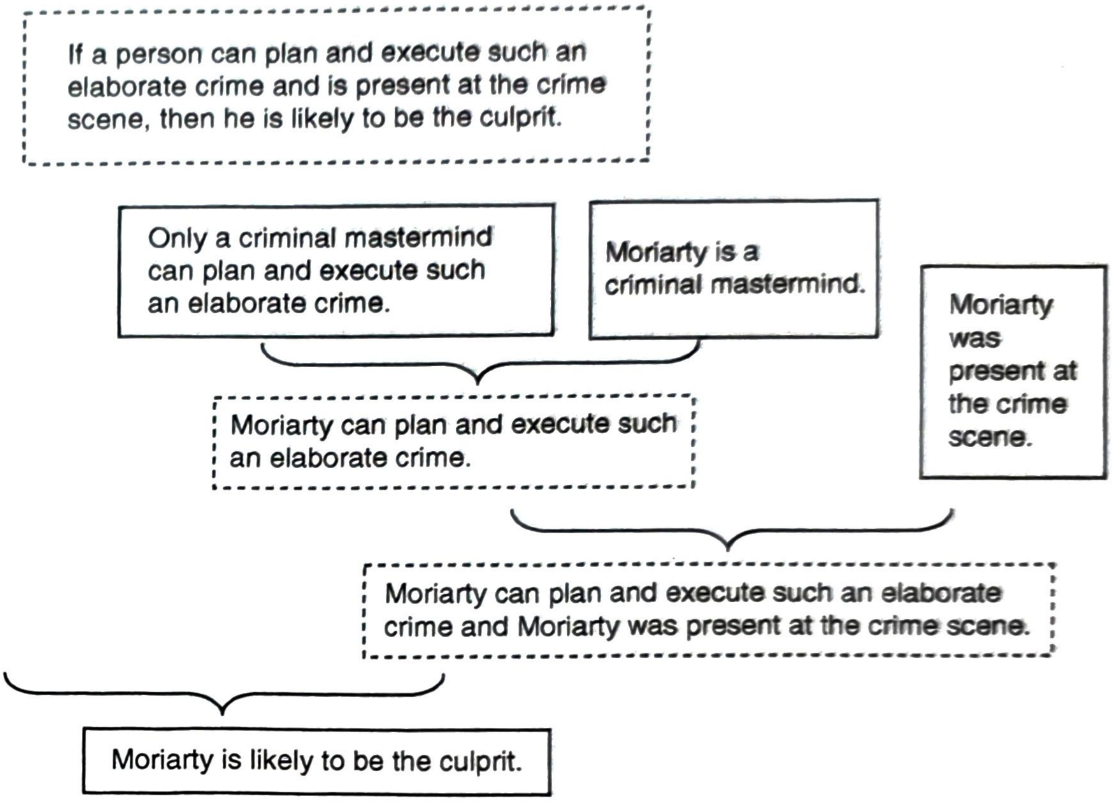
(Premise 3, implicit) A person is likely to be the culprit if he can plan and execute such an elaborate crime and he is present at the crime scene.

(Premise 4, implicit) Moriarty can plan and execute such an elaborate crime. (The conclusion of premise 1 and 2)

(Premise 5) Moriarty was present at the crime scene.

(Conclusion) Therefore, Moriarty is likely to be the culprit.

The argument structure can be represented by a diagram as follows.



Exercise 1.1

Identify the arguments in the following passages and write them into the standard form. Highlight the premise and conclusion indicators. Notice that sometimes our actual use of language may contain hidden assumptions or **implicit premises**. Also, sometimes ideas are framed in a question rather than a statement. Identify and rephrase them in explicit statement form whenever necessary.

1 A prince ... must imitate the fox and the lion, for the lion cannot protect himself from traps, and the fox cannot defend himself from wolves. One must therefore be a fox to recognize traps, and a lion to frighten wolves. (Niccolo Machiavelli (1532), *The Prince*)

2 Since happiness consists in peace of mind, and since durable peace of mind depends on the confidence we have in the future, and since that confidence is based on the science we should have of the nature of God and the soul, it follows that science is necessary for true happiness. (Gottfried Leibniz (1677), *Preface to the General Science*)

3 Liberty means responsibility. That is why most men dread it. (George Bernard Shaw (1903), *Marxism for Revolutionists*)

4 'And how do you know that you're mad?' 'To begin with,' said the Cat, 'a dog's not mad. You grant that?' 'I suppose so,' said Alice. 'Well, then,' the Cat went on, 'you see, a dog growls when it's angry, and wags its tail when it's pleased. Now I growl when I'm pleased, and wag my tail when I'm angry. Therefore I'm mad.' (Lewis Carroll (1865), *Alice's Adventures in Wonderland*)

5 If a man says, 'I love God', and hates his brother, he is a liar; for he that loves not his brother whom he has seen, how can he love God whom he has not seen? (The Bible, 1 John 4:20)

6 Try to identify an argument in daily newspapers, magazines, or blogs that you read.

1.3 Deduction and Induction

Apart from being able to identify arguments, we also need to be able to identify the type of argument involved. There are at least two main types of inferences. For example, compare the arguments in (6), (7) and (8). (6) operates very differently from (7) and (8) in that even if the premise of (6) is true, the conclusion is only *probably* true, rather than definitely true. In other words, the conclusion is likely to be true though it could be false even when the premises are true. We call this type of argument **induction.** However, for (7) and (8), if the premises are true, we know already whether the conclusion must be true or not. In other words, even though the premises may not actually be true, we can already decide whether the conclusion *would follow* or not. We can say that the truth of the premises *completely determines* the truth of the conclusion given the validity of the argument. This form of argument is called **deduction.**

Induction is commonly used in scientific inferences to generalize claims based on particular observations and experiments. Deduction is more often found in mathematics and philosophy because it promises knowledge with certainty from premises which do not need to already be known as true or false. This book will be concerned mainly with deduction.

We may contrast the differences between deduction and induction as follows:

Deduction	Induction
Achieves certainty: for a deductively valid argument, if the premises are true, then the conclusion must be true.	Achieves probability: if the premises are true, then the conclusion is likely to be true, although it does not have to be true.
Adding new premises may not lead to a stronger argument.	Adding new premises may alter the strength of the argument.
The conclusion is already contained in the premises; hence in a manner of speaking, no new knowledge is involved.	Increases empirical knowledge.

Key idea: Induction

Induction is an important branch of logic, though we are not going to discuss it much in this book. Most scientific discoveries, statistical analyses and generalizations are based on inductive reasoning. Inductive arguments are evaluated as strong or weak. Only deductive arguments are described as valid or invalid. All inductive arguments are in fact invalid.

1.4 Truth and validity

The main task of logic is not just to identify an argument but also to evaluate it. Two criteria are used: (i) the truth of individual statements (premises and conclusion), and (ii) the validity of the whole argument. Both criteria need some explanation.

TRUTH

Logic aims at truth. Indeed, all reasoning aims at it. **Truth** is whatever is the case. Intelligent beings like humans want to know what is out there, what the case is and what it is not, what connections there are between things, what has happened and what could have happened. All science aims at establishing such knowledge. Logic, in particular, can be regarded as a special kind of science, in that it is interested in not only what things are, but also what they could be. Hence,

although we may not know how things actually are, we do want to know *what could follow if* things were so and so. For example, look at the following argument:

Example (13):

(1) It is morally wrong to inflict unnecessary pain on other beings.

(2) Some animals can feel pain.

(3) Therefore, it is morally wrong to inflict unnecessary pain on some animals.

We may not already know the truth of the premises. In example (13), the truth of (1) is subject to moral debate; that of (2) is subject to scientific investigation. However, we do know that *if* (1) and (2) are true, *then* (3) will follow. We can tell this result from the relation between (1), (2) and (3). Such a relation between premises and conclusions is called **validity**.

Key idea: Truth and validity

An argument usually consists of several statements, i.e. a conclusion and some premises.

A Can a *statement* (a premise or a conclusion) be valid or invalid?
B Likewise, can an *argument* be true or false?

No, they can't. A statement is true or false; yet only an argument can be valid or invalid. It is because validity is a *relation* between statements, rather than a *property* of an individual statement by itself. Validity indicates whether the conclusion follows if the premises are true. Although we sometimes hear of people making 'valid' statements in ordinary language, this is in fact a mistake. Similarly, only a statement can be true or false. An argument contains statements which are true or false, but the argument itself is not true or false. If an argument is valid, then its conclusion must be true when the premises are true. However, we say the argument is valid, not that it is true.

VALIDITY

An argument is valid if and only if the conclusion follows the premises. That means, the conclusion cannot be false if all the premises are true. Any argument that is not valid is invalid.

Let us illustrate the concept of validity by examples. As we move on, we will introduce more formal skills in how to determine the validity of an argument. In fact, validity determination is one of the main concerns of logic.

Below is an intuitive valid argument:

> Example (14):
>
> (1) If it rains, then the ground is wet.
>
> (2) It rains.
>
> (3) Therefore, the ground is wet.

The argument is valid because the conclusion follows from the premises. However, the premises do not need to be true. (1) does not have to be true; it is just a contingent fact of our actual world. (2) is false, too, if it is uttered on a sunny day. However, this does not alter the fact that if (1) and (2) are both true, then (3) must be true.

Contrast this with another argument:

> Example (15):
>
> (1) If it rains, then the ground is wet.
>
> (2) It does not rain.
>
> (3) Therefore, the ground is not wet.

This is an invalid argument because even if the premises (1) and (2) are true, (3) can still be false. For example, someone may splash water on the ground, or break a water bottle accidentally and so the ground becomes wet, such that even when it does not rain, the ground may still be wet. The truth of the premises thus does not *guarantee* the truth of the conclusion; hence the argument is invalid.

MORE EXAMPLES

Determine by intuition whether the following arguments are valid.

Example (16):

(1) Confucius was a great teacher.

(2) There are seven days in a week.

(3) Therefore, $2 + 5 = 7$

In (16), the premises and the conclusion are all true. The conclusion is even necessarily true because mathematical truths do not depend on contingent matters in the empirical world. However, the premises are not relevant to the conclusion. So, although the argument can be called valid, it is only vacuously valid. And it does not represent good reasoning.

Example (17):

(1) All spiders are insects.

(2) All insects have eight legs.

(3) Therefore, all spiders have eight legs.

In (17), both (1) and (2) are false. Indeed, spiders are not insects and insects have six legs instead of eight. However, the argument is valid. It is because if the premises were true, then the conclusion would follow and be true. Contrast this with the following.

Example (18):

(1) All insects are spiders.

(2) All insects have eight legs.

(3) Therefore, all spiders have eight legs.

The premises of (18) are false; however, the argument is invalid, too. It is because even if (1) and (2) were true, it still does not guarantee that (3) is true. Premise (1) states that all insects are spiders. If this is true, the set of spiders is supposedly larger than that of insects. This follows that even if all insects have eight legs, the set of insects does not coincide with all spiders, so there still can be spiders that are not insects and so do not have

eight legs. Compare this with the following argument which has exactly the same form and it is easy to see that the following is invalid.

Example (19):

(1) All mothers are women.

(2) All mothers have children.

(3) Therefore, all women have children.

The construction of (19) in contrast with that of (18) illustrates a good strategy in arguing called **logical analogy**. It is a strategy in which the reasoner provides two arguments with the same argument form, one obviously valid or invalid while the other is not so obvious. By pointing out the validity of one argument, it shows how the other, although more obscure in outlook, shares the same logical properties.

SOME COMMON CONFUSIONS

▶ 'An argument with true premises and a false conclusion can be a valid argument'

Wrong. If both an argument is valid and all its premises are true, then the conclusion must be true.

A valid argument may have:

- true premises and true conclusion
- false premises and true conclusion
- false premises and false conclusion

A valid argument just can never have:

- true premises and false conclusion

▶ 'An argument with true premises and a true conclusion is always a valid argument'

Wrong. We do not determine validity based on the individual facts that the premises are true and the conclusion is true. Rather, validity is about the relation between premises and conclusion, that *if* the premises are true, *then* the conclusion

cannot be false. The premises and the conclusion have to be connected in the right way, regardless of whether they are true indeed. An argument can have true premises and a true conclusion, but still be invalid. Consider:

Example (20):

(1) No spiders are insects.

(2) All insects have six legs.

(3) Therefore, no spiders have six legs.

This is an invalid argument because the premises do not state that only insects have six legs. Perhaps some other types of animals have six legs, too, and spiders are one of them. So, although the premises and the conclusion of this argument are indeed true, the premises do not guarantee the truth of the conclusion and the argument is still invalid.

▶ 'An argument with one or more premises which are false could have a true conclusion and be a valid argument'

Correct. Consider the argument in example (14). It is possible that on a sunny day (making (2) false) the ground is spilled wet (making (3) true). Nevertheless, the validity of the argument does not change.

▶ 'Can there be valid arguments with false premises and a false conclusion?'

Yes, below is one example. The premise (1) is false; however, if it is true, then the conclusion (2) is true. So the conclusion does follow the premise.

Example (21):

(1) Two is greater than three.

(2) Therefore, two squared is greater than three squared.

EVALUATING AN ARGUMENT: VALIDITY, SOUNDNESS AND COGENCY

Now that we are armed with two concepts (truth and validity), we can evaluate arguments with these two criteria. We want

an argument to be at least valid, because if it is invalid, then the inference will not lead to any secure knowledge. On many occasions we do not really know whether a statement is true, yet with a valid argument, we know at least the relationship between the statements, i.e. that if something is so and so, then a certain consequence will follow. This knowledge is useful. For example, in mathematics, many people feel reluctant to assert that mathematical objects such as numbers exist as abstract entities. Yet no matter what they are, at least we know what consequence follows from accepting certain axioms (premises, assumptions). Knowledge of these relations is often sufficient to make the whole system function when solving real mathematical problems.

A better type of argument is indeed where the argument is valid and the premises are all true. We call such arguments **sound arguments**. They are better because validity guarantees that if the premises are true, then the conclusion must be true, and now all premises are true indeed so it follows that the conclusion of a sound argument is true, too. That is just exactly what we want: knowledge about truth with certainty.

Example (17) is a valid argument; however, it is unsound because its premises are false. In fact, even if only one premise is false, the argument containing it cannot be a sound argument. The following are examples of a sound argument:

Example (22):

(1) 289 × 312 = 90,168

<u>(2) If 289 × 312 = 90,168, then 289 is a factor of 90,168.</u>

(3) Therefore, 289 is a factor of 90,168.

Example (23):

(1) Every man dies.

<u>(2) Socrates is a man.</u>

(3) Therefore, Socrates dies.

Both arguments are sound indeed. However, not all sound arguments have premises that are evidentially true. We thus further distinguish sound arguments into two types: cogent

and not cogent. The argument in (22) is sound but not cogent whereas (23) is sound and cogent. (22) is not cogent because people may not have actually multiplied 289 by 312 and found out that it equals to 90,168. So, (1) is not evidently true. Moreover, the truth may not be easily accessible to some people (e.g. those who are innumerate). In contrast, all premises in (23) are obvious and evident. So (23) is much easier to accept.

I use the term '**cogent argument**' to refer to any argument (including inductive argument) whose 'premises are acceptable, relevant to and sufficient for its conclusion' (Johnson and Blair 1977). Such definition allows coverage of a wide range of inferences in everyday discourse. Hurley (2000) defined a cogent argument as 'an inductive argument that is strong and has all true premises'. As such, a cogent argument is the inductive equivalent of a sound argument. I prefer, however, to adopt a broader understanding rather than to limit the term to inductive arguments.

Let us see more examples. Both arguments below are sound; indeed, they have exactly the same argument structure. However, (24) is cogent while (25) is not.

> Example (24):
>
> (1) If the earth is flat, then the mast of a ship would not disappear over the horizon after leaving a port.
>
> (2) The mast of a ship does disappear over the horizon.
>
> (3) Therefore, the earth is not flat.
>
> Example (25):
>
> (1) If there is a finite number of prime numbers, then the product of those primes + 1 is not a prime.
>
> (2) The product of those primes + 1 is a prime.
>
> (3) Therefore, there is not a finite number of prime numbers.

Cogency reflects the persuasive power of an argument. Since it is a matter of knowledge, it is also relative to context and its intended audience. One may find certain premises evidently true while others not. For example:

> Example (26):
>
> (1) Whales are mammals.
>
> (2) Whales jump.
>
> (3) Therefore, some mammals jump.

This is a cogent argument nowadays, though it was not so in the past when whales were not generally recognized as mammals, but thought to be fish.

To sum up, we can define the three criteria of good arguments as follows:

- **Validity:** An argument is valid if and only if its conclusion is true whenever all its premises are true.
- **Soundness:** An argument is sound if and only if it is a valid argument and all its premises are true.
- **Cogency:** An argument is cogent if and only if its premises are acceptable, relevant to and sufficient for its conclusion.

Spotlight: Basic notions in logic

This chapter introduces several basic notions in logic.

- Logic is the study of good and bad reasoning. Reasoning is presented in arguments, which take the form of a conclusion supported by premises.
- A statement or a proposition is either **true** or **false**.
- An argument is evaluated in the following order: validity, soundness and cogency.
- A **valid** argument is one which, if all premises are true, then the conclusion cannot be false.
- A **sound** argument is a valid argument in which all premises are true.
- A **cogent** argument is a strong argument whose premises are evidentially true.

Exercise 1.2

1 Determine whether the following statements are true or false.

i A valid argument always has a true conclusion.

ii A sound argument may have a false conclusion.

iii A cogent argument always has true premises.

iv An argument with true premises and conclusion is a valid argument.

v An invalid argument may have a true conclusion.

2 Match the following terms with appropriate definitions.

i Argument	a An unstated premise
ii Assertion	b A structure that guarantees a true conclusion if the premises are true
iii Premise	c A structure comprised of reasons leading to a conclusion
iv Conclusion	d A statement held true with or without providing any reasons or supporting evidence
v Implicit assumption	e A belief that is formed without considering evidence for or against it
vi Valid argument	f A valid argument with true premises
vii Sound argument	g A statement derived from premises and from which it follows
viii Unsupported argument	h A statement from which an argument's conclusion is derived

1.5 Some additional remarks

This section aims to remove some common psychological barriers in learning logic.

IS LOGIC MATHEMATICS?

If you think mathematics is doing many calculations, measurements and counting, then no, logic is not mathematics. Take for example the game Sudoku, which seeks to fill a 9 × 9 grid with numbers 1–9 such that each column, row and 3 × 3 square

contains all digits with no repetition. It is a logical game but not a mathematical one because the whole game can be performed without using numerals. The game is about manipulating symbols according to certain rules and patterns, and any set of symbols can fulfil the same task. (Indeed, I have seen a sudoku game in currency signs, intended to appeal to workers in the City of London.) This shows that logic has a wider application than mathematics because logic is not restricted by actual meanings of the symbols used.

However, logic and mathematics do have profound similarities in other aspects, such as the involvement of abstract thinking and operations. A German logician Gottlob Frege (1848–1925), often regarded as the father of modern logic, led an ambitious project called **logicism** to reduce arithmetic completely to logic. The task was to define numbers in terms of logical operations alone, such that arithmetic would become completely general and no substantial mathematical content would be necessary. Frege (1884) did this by claiming that zero is that which belongs to the concept of 'being not identical with itself' (sec 74). Since nothing is not identical with itself, the concept of 'not identical with itself' thus refers to nothing, i.e. having zero element in the set of objects that satisfy the criteria of being identical with itself. So we can now define zero as the set of objects falling within the concept of 'being not identical with itself'. Once we get zero, we can define other numbers in terms of it. For example, the number one can be defined as the concept of 'being identical with zero' because there is only one concept which refers to zero, i.e. 'being not identical with itself'. The number two is that which has the same reference as the concept of 'being identical with zero or one', and so on.

The project was elegant and fundamental. However, Bertrand Russell (1872–1970) pointed out in 1901 a paradox (later coined Russell's paradox) of this project, and defeated it. This paradox concerns the idea of an empty set and its power set. An empty set is a set that has no element within it; a power set is a set made of sets. If we construct a power set containing an empty set, intuitively the empty set will become an element of itself. So the set of an empty set is not empty. Yet an empty set, by definition, should have no element. It thus seems that we do get something

2

Meaning

In this chapter you will learn about:

- ***the formal character of deductive logic***
- ***common types of unclear expressions***
- ***what meaning is***
- ***definition***

> *'When I use a word,' Humpty Dumpty said in rather a scornful tone, 'it means just what I choose it to mean – neither more nor less.'*
>
> *'The question is,' said Alice, 'whether you can make words mean so many different things.'*
>
> *'The question is,' said Humpty Dumpty, 'which is to be master – that's all.' [...]*
>
> *'When I make a word do a lot of work like that,' said Humpty Dumpty, 'I always pay it extra.'*
>
> Lewis Carroll (1871), *Through the Looking Glass*, Chapter 6

2.1 The formal character of deductive logic

Deductive logic does not depend on the actual meaning of individual words. Look at the following sample arguments. Do you think they have anything in common? If so, what is it?

> Example (1): If the Tories win, then there will be more budget cuts. The Tories will not win. Therefore, there will not be more budget cuts.
>
> Example (2): If you bake a cake for too long, then you ruin it. You have not baked the cake for too long. Therefore, you have not ruined it.

The two arguments do not have the same content: one is about politics, and the other is about baking a cake. The meaning of the words composing them is different, too. However, it seems obvious that they do have something in common. We can almost intuitively see that the first argument is not valid. It is because there may still be budget cuts even if the Tories lose, for example if there is a severe economic downturn. Similarly, there are also ways other than overbaking that ruin a cake, such as putting in the wrong ingredients. Hence, the satisfaction of the first and second premises does not guarantee

the truth of the conclusion. In fact, the way that such a guarantee is not met comes from exactly the same pattern. Let us strip the pattern out by replacing individual words with arbitrary symbols.

> Let A stand for the sentence 'the Tories win', B stand for 'there will be more budget cuts'. The first argument becomes:
>
> (1') If A, then B. Not-A. Therefore, not-B.

Similarly, for the second argument, let A stands for 'you bake a cake for too long' and B stands for 'you will ruin it'.

> (2') If A, then B. Not-A. Therefore, not-B.

(1') and (2') are exactly the same. Hence, we can see that beneath all the difference in meaning of the sentences, the two arguments actually have exactly the same form! It is this argument form which determines whether the argument is valid. We can give an intuitive but informal explanation as follows. 'If A, then B' determines what happens when the antecedent, i.e. A, obtains. However, it does not cover all the cases when A does not obtain. That means B may occur because of something else, not just because of A. Thus merely having the premises of 'If A then B' and A is not the case, we still cannot guarantee that B must not be the case.

Although meaning does not determine the validity of an argument, meaning does matter in logic in another way: we cannot identify the relevant argument form if we are not sure what sentences in an argument mean exactly. Examples (1) and (2) are clear in meaning. However, there are cases in which individual words may have multiple or obscure meanings. Or, there may be multiple ways to read the sentence structure, making it hard to articulate the correct interpretation of a sentence. If situations like these occur, depending on how we 'cash in' the meaning of the expressions in context, there may be different interpretations of an argument form.

2.2 Common types of unclear expressions

To heighten our awareness of meaning problems, let us identify several common types of unclear expressions. The principle is that in order to subject an argument to proper logical analysis, the argument must first have a clear and definite meaning as far as possible. Look at the following examples. Are these sentences clear and definite in meaning?

Example (3): Brian goes to the bank every day.

Example (4): There is no victor in war.

Example (5): Time flies like an arrow, fruit flies like a banana.

Example (6): Peter attacked the woman with a knife.

AMBIGUITY

The above examples may mean multiple things, so they do not have definite meanings. (3) can be taken to mean two situations – (3a) Brian goes to the embankment of a river every day, or (3b) Brian goes to a financial institution where people deposit and withdraw money every day. Here the word 'bank' has two possible meanings: 'river shore' or 'a financial institution'. When an expression can be interpreted to have more than one meaning, we call that expression **ambiguous.** Ambiguity is thus the first type of problem in meaning.

Ambiguity can come in many forms and have different causes. We can easily look in a dictionary and find that an individual word such as 'bank' has multiple meanings. However, in some other cases it is more covert. (4) is ambiguous not necessarily because there is a stalemate in a war in which no side can claim to have won or be defeated. Rather, one may assent to (4) because even if there is a winner, the price of winning may be so high that it causes great suffering to the winning country, too. Hence, in the end the best option is not to have wars at all.

(5) and (6) are ambiguous for different reasons. They concern not the meaning or criteria of fulfilling the sentence, but the grammatical structure. In (5), an expression is used as different parts of speech. In the first half of the sentence, 'flies' is a verb and the sentence means that time passes quickly. In the second half, 'flies' is part of a noun; the sentence means that 'fruit flies' are attracted to a banana. In (6), there are multiple ways to 'dissect' the sentence, too. The phrase 'with a knife' can be used to qualify Peter or the woman. If it qualifies Peter, it means Peter used a knife to attack a woman. If it qualifies the woman, it means Peter attacked a woman who carried a knife.

VAGUENESS

An expression is **vague** if it has a core meaning but does not have a clear boundary. Unlike ambiguity, vagueness is not about referring to completely different things, but is about matters of degree relative to a standard or a comparison class. Look at the following examples.

> Example (7): Woe is me! I am too old for a happy life.
>
> Example (8): Bald men are sexy.
>
> Example (9): An average 13-year-old boy is short.

In (7), 'old' is a vague expression, so is 'happy'. Whether a person is too old depends on the context and the purpose of the claim. There is no absolute definition of old age. If one is willing, one can live for long years and still not consider oneself old. Happiness is another vague concept, and it is also ambiguous. It is ambiguous in the sense that different things may constitute a happy life: one person might be happy if he marries a beautiful woman; while another person might be happy if she reads a good book, etc. Yet 'happiness' is also vague because there are degrees of happiness. I may be very happy to receive a free lunch when I am hungry, but the satisfaction would decrease if I have to take a second or even third lunch in one sitting. There are no definite answers to how much happiness a life has to have in order to be called a happy life either.

In (8), baldness is vague, so is being sexy. One loses one's hair gradually. While having zero hair is apt to be called bald and having, say, 10,000 strands of hair is not bald, it is difficult to say by what number of strands of hair one would step from being non-bald to being bald. Is it 2601? If so, will a person with 2600 or 2602 strands of hair belong to a completely different category, even though the difference in number of hairs is just minor? We can hardly justify any answer. This constitutes the so-called **sorites paradox** (see box below), which exactly addresses the problem of vagueness. A **paradox** is a puzzle that has apparently reasonable assumptions yet generates an absurd consequence. There is still not a satisfactory solution to the problem of vagueness.

Spotlight: The sorites paradox

The sorites paradox is also called the paradox of the heap. It is a paradox about vagueness. It runs as follows.

One grain of sand does not form a heap. Adding one grain still does not make a heap. Indeed, intuitively no one grain of sand is responsible for changing the status of the existing sum into something completely different. However, if someone keeps repeating the process of adding one grain of sand at a time over and over again, then eventually a heap will be formed. So, when does the heap come into being and what is the boundary between a heap and not a heap?

Several directions for handling vagueness are discussed in philosophy. These include the epistemological approach, the appeal to degrees of truth, and supervaluation. Interested readers may consult further references such as Sainsbury (2009) or Clark (2012).

(9) is a nice demonstration of how judgement of vague concepts depends on, or is relative to, the associated comparison class. An average 13-year-old boy is likely to be too short to be a professional basketball player but too tall to play on bouncy castles. So whether he is short or tall depends on what standard of comparison is used. Most ordinary use of language is not explicit in specifying the standard or the comparison class though.

Conversationists have to pick up the relevant standard in context. In (9), we also have to pick out in context what 'average' refers to: for example, it should refer to the height, rather than, say, intelligence. This is because the comparisons are all about height.

By now, you should be aware that vagueness is indeed a very common phenomenon in ordinary language use. Most ordinary expressions are vague. This does not mean, however, that they are all problematic and can't be used in reasoning or logical inference. We just need to decide whether a sentence's meaning is clear enough for evaluation in the relevant context.

OBSCURITY

An expression is obscure when it lacks a core meaning. With vague expressions we are not able to draw a clear boundary in all cases between, say, what is bald and what is not bald, or what is old and what is not old. Yet a competent language user still knows more or less what baldness or old age means; for instance, a person with no hair at all is certainly bald and a person at 100 years old is certainly old. So, a vague expression does have a core meaning. However, an obscure expression is different in that it does not even have a core meaning. Consider the following examples.

> Example (10): Everything counts.
>
> Example (11): You are always right to some extent (or, in some sense).
>
> Example (12): There's something in it.
>
> Example (13): It is an interesting argument…

Each of these examples seems to have said something. Yet no one can know what it is if we take the utterances literally. Take (10) as an example: 'Everything counts'. Yet what is 'everything'? Counts for what? We may hear people say, 'You are right in some sense' or 'You are right to some extent'. Yet to what and which extent exactly? In some sense, but what sense? Without further explanation, the speaker may mean completely different things. Worse, perhaps the speaker may just want to dismiss the conversation and make the hearer feel as if he is right and then shut up. So the speaker does not really mean that

the audience is right at all. A similar thing happens with (12) and (13). In the case of (12), that 'something' is not specified. In the case of (13), 'interesting' can be a vague expression when the speaker really finds the argument interesting to some degree. It can also be used to imply that the speaker is not interested in the argument at all, but just says something polite to dismiss the discussion.

INCOMPLETENESS

An incomplete expression is an expression in which the domain is not specified. Its reference is often sensibly fixed only by making some contextual adjustment.

> Example (14): Let's go to the pub afterwards.
>
> Example (15): Everyone is happy.

The definite article 'the' is used in English to refer to a singular. Philosopher and logician Bertrand Russell (1905) told us that phrases taking the form of 'the X' should be analysed as denoting something that uniquely satisfies the property of X in the whole domain of discourse, namely, the set of objects in the whole universe. For example, 'the discoverer of the relativity theory' denotes no one but Albert Einstein; 'the smallest prime' denotes the number 2 because it and it alone satisfies the criteria of being a prime and being the smallest among all primes.

However, if this is the case, (14) would not make sense because obviously there is not just one place in the whole universe which is a pub and to which the interlocutors will go after making the utterance. There are thousands and thousands of pubs in existence, and that have ever existed in history. It thus seems a better strategy to consider 'the pub' as an incomplete expression – it does not mean there is 'exactly one pub in the whole universe' but there is 'a pub in a particular time and space'.

The context of the conversation usually provides some clue to qualify the domain in cases of incomplete expressions. For example, the interlocutors may be referring to a pub that they went to together before, or the name or address of the pub

may have been mentioned before and so 'the pub' simply refers to the place previously mentioned. In the latter case, 'the' is used like an anaphor.

Similarly, with (15), the world is inhabited by billions of people. It is thus extremely unlikely that literally every one of them is happy at any one moment. Yet when people utter this sentence, they are probably not talking about the whole universe anyway, but only a particular group in a particular place, such as a party the speaker is at. 'Everyone' is thus an incomplete expression short of 'everyone in x (a certain time and place)'.

IDIOSYNCRASY

Idiosyncratic expressions are jargon terms or expressions with unusual usage deviating from ordinary usage without explicit definition. Experts and professionals use jargon all the time within their communities and it is often necessary to do so in order to communicate exact meanings. Sometimes secret codes are used for communication among a select group of people to keep things to themselves; these codes are also regarded as idiosyncratic expressions. Idiosyncrasy is not necessarily a bad thing. It all depends on the purpose and the context of communication. For example, the Humpty Dumpty lines quoted at the beginning of this chapter are but a funny joke. The story goes as follows.

> 'I don't know what you mean by "glory",' Alice said.
>
> Humpty Dumpty smiled contemptuously. 'Of course you don't – till I tell you. I meant "there's a nice knock-down argument for you!"'
>
> 'But "glory" doesn't mean 'a nice knock-down argument", Alice objected.
>
> 'When *I* use a word,' Humpty Dumpty said, in rather a scornful tone, 'it means just what I choose it to mean – neither more nor less.'

Although idiosyncrasy is not necessarily a defect, it may cause problems if it is used in such a way that causes communication breakdown or confusion in meaning, or when the new meaning

is not properly and sufficiently explained. Look at the following example.

> Example (16): No one is free because we can't even choose when and where we are born.

The speaker uses 'free' in some atypical sense. In political philosophy, we regard ourselves as free as long as we are not subject to intervention from other persons or institutions. We do not complain about the existence of natural constraints such as 'we can't breathe without oxygen' as a deprivation of freedom. Similarly, in ethics, intuitively we regard ourselves as free and morally responsible for our actions if we could have decided to act otherwise. When and where a person is born is a matter of nature; it should not be considered as rendering one not to be free in either the political or the moral sense. Of course, the speaker of (16) may insist on a special usage, or even intend it as a joke. However, if we are to take it seriously, the speaker would have the cognitive burden to explain clearly what it means and how it relates to the polemic issue at hand.

A related issue is the use of euphemisms. As said above, experts and professionals use jargon all the time. However, they must still be cautious about whether it is justified to use so many of them, especially when they are trying to introduce new ones. In example (17), a new term 'post-truth democracy' was introduced, yet nowhere in that article has the author explained what it means. He just went on to make an important claim about whatever this term refers to as being not a democracy. What is the point of introducing a new term without explanation or definition? Can the whole argument go through without using the new term?

> Example (17): A 'post-truth democracy', such as the *New York Times* saw on the horizon during the last Presidential elections, would no longer be a democracy. (Jurgen Habermas (2006) 'Religion in the Public Sphere', *European Journal of Philosophy* 14(1): 1–25)

EMOTIVE WORDS

Emotive words are expressions that would usually arouse particular feelings or judgements. Like idiosyncrasy, evaluative expressions, or 'loaded' words, are not necessarily bad. Sometimes we want to express our emotions through words, and not just report an objective state. However, there are subtle cases in which the audience may or may not be aware of the emotion or evaluation attached to words and be misled as if they were just describing facts. Can you identify the different implications in each pair of the phrases below?

Example (18): 'Stubborn' vs 'perseverant' or 'strong-willed'

Example (19): 'Leave me alone' vs 'get lost' or 'piss off'

Example (20): 'Revenue enhancement' vs 'tax increase'

Example (21): 'Tough choices' vs 'budget cuts'

Example (22): Hate language: 'black people' vs 'niggers', 'homosexuals' vs 'faggots'

In the next chapter, we will explore fallacious reasoning caused by abuse of language.

Exercise 2.1: Spotting problems of meaning

Identify any unclear expressions in the following. Explain what type of problem each of them has and why.

1 Success is for those who do the right thing at the right time.
2 Veterans don't die, they just disappear.
3 Vanity of vanities! All is vanity. (The Bible, Ecclesiastes, 1:2)
4 Be good and good things will come to you, eventually.
5 No smoking seats available.
6 Honesty is the best strategy.
7 Western democracy is not real democracy. We strive for another type of democracy, the real one.
8 I am my world. (The microcosm.) (Ludwig Wittgenstein (1921), *Tractatus Logico-Philosophicus* 5.63)

2.3 What is meaning?

PROPOSITION, SENTENCE AND STATEMENT

So far, we have been using the terms 'proposition', 'sentence' and 'statement' almost indiscriminately. However, there are indeed differences between them and such differences do carry philosophical significance. The contrast is particularly sharp between proposition and sentence. Let us discuss them in detail.

'Proposition' is a logical notion. A **proposition** is anything the content of which is capable of being true or false. A proposition is an abstract entity. A **sentence** is a string of symbols that is complete in grammatical structure and meaning. So, in contrast, 'sentence' is a linguistic notion. Finally, a **statement** is something that expresses a belief. It is a matter of the state of mind.

Proposition and sentence are not necessarily the same. A sentence may express a proposition. Indeed, different sentences can express the same proposition if they convey the same content. However, it is also possible that a sentence does not express any proposition at all, or that a sentence expresses different propositions in different contexts. Let us consider some examples.

Example (23): Snow is white.

Example (24): La neige est blanche.

Example (25): 雪是白的。

Example (26): 雪は白です

Each of the above examples is a sentence. A sentence is a string of symbols that forms a grammatical structure and is meaningful on its own. It is a basic unit of meaning. Different languages have different sentences because, obviously, the symbols (words or vocabulary) are different and the grammatical structure may be different, too. For instance, Japanese has the verb at the end of a sentence while English has it between the subject and the predicate. However, each of these sentences in (23)–(26) expresses the same content of meaning, namely that snow is white. So they do express the same proposition: Snow is white. Note, for our purposes here, when we put an expression in

Proposition → true / false
Sentence → utterance (no true or false)
Statement → belief, idea, viewpoint (true for me, but may not for other)

quotation marks, it indicates the linguistic use of it, namely the symbols and the sentence, etc. However, if no quotation marks are used, we refer to the content instead.

> Example (27): 'Snow is white' expresses the proposition that snow is white.

> Example (28): 'La neige est blanche' expresses the proposition that snow is white.

tầm thường, k' đáng kể

It may seem trivial that we write (27). However, it is not trivial really. The string of symbols in quotation marks is a sentence. If we replace another sentence of the same meaning as in (28), the sense of triviality would be gone.

Different sentences in the same language can also express the same proposition. For instance:

> Example (29): London is situated south of York.

> Example (30): York is situated north of London.

> Example (31): I am hungry (uttered by Dr Lauben).

> Example (32): Dr Lauben is hungry.

(29) and (30) represent the same fact, though they describe it from a different perspective. If (29) is true, namely that London is situated south of York, then (30) must also be true, namely that York is situated north of London. (29) and (30) share the same truth-value. This illustrates that indeed one of the necessary conditions for two expressions to be the same proposition is that they always share the same truth-value.

(31) and (32) are also arguably the same proposition expressed in different forms of speech: (31) represents a first-person point of view whereas (32) is third-person reported speech. I say they are *arguably* the same. The arguable point is that while philosophers do not dispute that (31) and (32) represent the same fact and thus are true or false together, some argue that the term 'I' brings a different mode of presentation from 'Dr Lauben'. Everyone has a private privileged access to him or herself, but not to others. Hence, when we say 'I', what is presented in our mind is more direct and intimate than when we say our own names in a third personal voice. Gottlob Frege

(1918) considered that 'I' and 'Dr Lauben', though they have the same reference, do have different senses. There is no consensus yet, however, whether two expressions need to be identical in both sense and reference in order to be the same proposition.

Example (33): I am hungry.

(33) is a sentence. It does not express any proposition unless it is uttered in a context. We can say that a sentence does not by itself express anything. So indeed it is utterances that count. Some utterances, such as those involving demonstratives and indexical expressions like (33), are more context sensitive than others. Depending on the context, utterances will express different propositions. For example, uttered by Wallace, 'I am hungry' expresses the content of Wallace is hungry. However, uttered by Tommy, the same sentence would express the content of Tommy is hungry.

There are other cases in which a sentence does not present a proposition at all. Questions, commands, exclamations, expressions of emotions, etc., are not themselves true or false; hence they are not propositions. Yet they could be expressed in the form of sentences, especially if we take sentences broadly to mean any strings of grammatically well-formed symbols.

Conversely, a proposition does not have to be represented by a sentence either. For instance, 'Nonsense!' is not a sentence, but it expresses a belief and the proposition that a certain thing is nonsense. Similarly, expressions such as 'Fire!' or 'Water!' may express complete thoughts in context, though they have only one word and are not complete sentences.

To sum up, we should now be more aware of the difference between proposition and sentence. Logic is concerned with propositions. However, since propositions are usually expressed in sentences, we also need to analyse and clarify the meaning of sentences alongside our discussion of logic.

VARIOUS MEANINGS OF 'MEANING'

In ordinary language, we often just talk of meanings of sentences or expressions. Yet what we mean by 'meaning' could mean many things (pun intended). This section introduces some

discussions in the philosophy of language about what meaning is. Several distinctions are made to sharpen our understanding. I also briefly discuss word meaning versus sentential meaning. This part is quite philosophical. It is fine to pass if you are not interested in it.

▶ (i) Sense and reference

There are three basic functions of language:

1 Informative function: to communicate information

2 Expressive function: to express emotions, feelings and attitudes

3 Directive function: to cause or prevent overt action.

To accomplish any of these functions, language has to have a representative role, i.e. we can use language to talk *about* things in the world (objects, events, state of affairs, etc.) and thoughts in our mind. The relation between language and the world is reference.

A referent is the thing an expression stands for. An expression may stand for singular objects or a set of things. For example, 'Barack Obama' stands for a single person Barack Obama. 'Black people' stands for a big group of people including Barack Obama, Nelson Mandela, Martin Luther King, Will Smith, Michael Jackson… The reference of an expression is also called its **extension** or **denotation**.

But is meaning just the same thing as reference?

An expression does not just stand for certain object(s); often, it also stands for them for a reason. For example, there is a reason why the term 'black people' denotes Barack Obama, Nelson Mandela… but not Bill Clinton, Tony Blair, Queen Elizabeth II... The former group share some common properties while the latter do not, such as having dark skin and ethnic origins traced back to Africa or the Caribbean. Indeed, these common properties determine who else would fall within the set of objects denoted by the expression. We can say that an expression denotes certain object(s) *in virtue of* the property so expressed. John Stuart Mill (1872) called the property the **connotation** of an expression. To use his example, the denotation of 'man' is Peter, John, Mary… and the connotation of 'man' is humanity.

Mill believed that proper names have no connotation but only denotation. However, Gottlob Frege (1892) famously argued that even proper names have something associated with them. He used the example: 'Hesperus is Phosphorus'. Hesperus is the name of the heavenly body which first becomes observable in the evening, also known as the evening star. Phosphorus is the name of the last remaining heavenly body observable in the morning, also known as the morning star. It took many years for astronomers to discover that Hesperus is indeed Phosphorus, that they are both Venus. So although the names 'Hesperus' and 'Phosphorus' refer to the same object, it is *not known* by the meaning of the words that they so refer. From this Frege concluded that their cognitive values must be different. To grasp the meaning of an expression would require more than just knowing its reference. Meaning, therefore, must consist of something more than reference. Frege calls this something else **sense**.

Rudolf Carnap (1956) formalized sense into a logical function called **intension**. He did so by introducing modality, the notion of possible worlds. A logical function is like a machine mapping one value to another. Extension is a function (machine) that leads from the words to the objects within any given possible world. Whereas intension is a function that leads from different possible worlds to the objects. For example, 'a creature with a heart' refers to the same thing as 'a creature with a kidney' in the actual world; so they have the same extension. However, we can imagine in some other possible worlds, parallel universes in which things such as the laws of nature are different, and a creature with a heart might not have a kidney. So the intension of 'a creature with a heart' is different from that of 'a creature with a kidney'. This difference seems to capture the difference in meaning. So it is claimed, meaning consists of not only extension but also intension.

To sum up, most philosophers agree that meaning is at least reference, denotation or extension. However, they disagree on whether meaning is only reference, denotation or extension. Some propose other semantic notions such as sense, connotation or intension. It may not be easy to explain clearly and exactly

what these notions are. For instance, some commentators argue that even Frege himself did not spell out clearly what senses are. For our purpose, let us just leave these issues aside but recognize the many layers of things that are called meaning.

My point remains: to clarify the meaning of an expression we sometimes need to clarify its sense, sometimes its reference, sometimes both.

Spotlight: Meaning

Mill: Denotation versus connotation

Frege: Reference versus sense

Carnap: Extension versus intension

(ii) Analytic and synthetic statements

Apart from sense and reference, another concept useful for understanding meaning is the distinction between analytic and synthetic statements.

An **analytic** statement is a statement which is true or false by virtue of its meaning alone. Given a fixed interpretation, an analytic statement once is true (or false) is always true (or false). A statement which is always true is called a **tautology**. A statement which is always false is called a **contradiction**. Analytic statements are therefore often paired up with the categorization of tautologies and contradictions. For example:

Example (34): Bachelors are unmarried. (tautology)

Example (35): Triangles have four angles. (contradiction)

(34) is true but we do not need to ascertain this truth by going out searching for bachelors in the world and asking them whether they are unmarried. On the contrary, we only need to look up the meaning of the word 'bachelor', which is an 'unmarried man'. Unmarried men are of course unmarried. Therefore, if we grasp the meaning of the statement, we know already that it is true and is always true.

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Similarly, (35) is always false and its falsity is secured by virtue of the meaning of the statement, too. It is because 'triangles' simply means three-angled geometric figures. If one grasps the meaning of all the key words in the sentence, one already knows that it is false.

A synthetic statement is any statement that is not analytic. The following are examples:

> Example (36): The Sun rises in the East.
>
> Example (37): All bodies are heavy.

The truth or falsity of a synthetic statement cannot be assured simply by virtue of its meaning. Rather, we need to look into reality, find out facts concerning the claim in order to decide whether the statement is true or false. It implies that synthetic statements are true or false by empirical investigation. Its truth is contingent, depending on the actual world we are in, rather than just being logically or conceptually true.

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In sum, analytic statements are often regarded as ***a priori*** (do not require empirical justification) and **necessary** (always true or always false) whereas synthetic statements are ***a posteriori*** (require empirical evidence) and **contingent** (true in some worlds but can be false in others).

Spotlight: A classical view on the nature of propositions

Analytic: *a priori*, necessary

Synthetic: *a posteriori*, contingent

Immanuel Kant (1724–1804) is the philosopher mainly responsible for the above associations, although he argued for exceptions too. For instance, he believed that moral truths are synthetic but *a priori* (i.e. they are not found to be true by analysing the meaning of words, nor do they require empirical evidence). Later on, some philosophers like W. V. O. Quine (1953) questioned whether there is a clear and principled distinction between analytic and synthetic statements. Saul Kripke (1980) also argued that there are necessary *a posteriori*

statements (e.g. 'water is H_2O') and contingent *a posteriori* statements (e.g. 'the stick stipulated to be the standard metre is one metre long').

The methodological point for making the analytic/synthetic distinction is that we can decide the truth and falsity of some statements simply by clarifying their meaning. It shows the significance of conceptual analysis. Historically, logical positivists such as A. J. Ayer (1946) even go so far as to claim a neat and *a priori* determination of the meaningfulness of a statement by examining whether it is analytic or synthetic and, if it is synthetic, whether it can be verified or falsified. This thesis has been seriously challenged, but conceptual analysis is still a useful tool in sharpening reasoning and argument.

2.4 Definition

There are many ways to clarify meaning. Some usual methods include specifying the context, providing paradigmatic examples, tracing the origin of words, clarifying speaker intentions, etc. Giving a clear and precise definition of the key terms is also one of them. This section focuses on definition and discusses some common types. We also look at criteria to distinguish good definitions from bad ones.

THE STRUCTURE OF A DEFINITION

A definition is composed of two parts. A ***definiendum*** is the term to be defined, and a ***definiens*** (this is the singular form though it ends in 's'; the plural form is *definientia*) is that which defines the term. We symbolize the relation of definition by a logical notation: '$=_{df}$'. For example:

> Example (38): 'Triangle' $=_{df}$ 'a closed geometrical figure with three straight lines and three angles'

The left hand side of the notation of definition, i.e. 'triangle', is the *definiendum* whereas the phrase on the right hand side, namely, 'a closed geometrical figure with three straight lines and three angles', is the *definiens*.

TYPES OF DEFINITION

There are many ways to define an expression. We have already discussed that meaning can be extension or intension. Similarly, we can also categorize definitions in these two directions. A denotative definition specifies the extension of an expression, i.e. the set of objects that fall under the *definiendum*. A connotative definition spells out its intension, i.e. criteria by which an object counts as belonging to the set specified by the *definiendum*. Here we introduce three types of denotative definitions and three types of connotative ones. The following diagram marks our classification explicitly.

- Definition
  - Denotative
    - Enumeration
    - Ostensive definition
    - Recursive definition/inductive definition
  - Connotative
    - Synonymous definition
    - Operational definition
    - Definition by genus and difference

▶ 1 Enumeration

Enumeration cites examples of objects that fall under the *definiendum*. Sometimes individual objects are stated (as in (39)); sometimes sub-classes are used (40). The following are examples:

> Example (39): 'Skyscrapers' $=_{df}$ Empire State Building in New York, Burj Khalifa in Dubai, Petronas Twin Towers in Kuala Lumpur, Bank of China Building in Hong Kong…
>
> Example (40): 'Vertebrates' $=_{df}$ fish, amphibians, reptiles, birds, mammals

Enumeration has the advantage of *directly* pointing out objects in the world which fall under the name. Note that on the left-hand side, the *definiendum* is put in quotation marks signalling that it is a linguistic item; however, on the right-hand side no items are put in quotation marks because it is not the names that we are referring to, but the actual objects.

Enumeration gives a very concrete and straightforward explanation of the meaning of the word. However, it also has disadvantages. For one, a *definiendum* may stand for a very large, if not infinite, plurality of objects yet counting is seldom complete and exclusive. Thus, there are many skyscrapers in the world and there will be infinitely more in the future. There are only a limited number of sub-classes of vertebrates so we can exhaust them; however, we can still hardly enumerate all species within each sub-class. The audience is expected to draw similarities from the objects or classes mentioned and 'go on with their own' to apply to other objects. Philosophically, this may cause a problem because one may always argue that a certain application is wrong no matter how many correct instances were cited before. Ludwig Wittgenstein (1958) called this a paradox of rule-following (see the Spotlight box below). The problem applies to not just enumeration but also generally all rule-governed activities including connotative definitions. The Spotlight box will explain why this is so too.

Spotlight: The rule-following paradox

Suppose a teacher asks a pupil to add two in the number series. So the pupil counts: 2, 4, 6, 8, ... 998, then he jumps to 1002, 1006, 1010, etc. The teacher stops him and says it was wrong. The pupil asks, 'Why is it wrong? I thought this is exactly what the rule means.' Was the pupil really wrong? On what basis do you make such judgement?

The problem is this. Rules are supposed to apply to instances. Yet for a rule to be a rule, there must always be some instances to which it has not been applied yet. What action would count as following the rule in such cases? Or more exactly, what rule are we talking about? What is the difference between 'do X' and 'do X until Y', 'do X except Z', etc., when Y and Z etc. are yet to happen? Are there differences and what justification is there to determine which action counts as following which rule?

Wittgenstein's conclusion: 'This was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule. The answer was: if everything can be made out to accord with the rule, then it can also be made out to conflict with it. And so there would be neither accord nor conflict here.' (Wittenstein (1958), section 201)

Saul Kripke (1982) interpreted Wittgenstein as being a sceptic against meaning. The case is significant because whatever we think of as rule-governed activities are not actually *governed* by the rules. It is we who decide to act in one way or other. It is we who decide a rule should be interpreted in this way or another, if any. A rule does not by itself say anything or lead us in any direction.

▶ 2 Ostensive definition

Although enumeration is a fairly direct way of showing the meaning of a word, it actually still presupposes a certain cognitive background. For example, someone who has never seen or heard of any of the buildings (even their pictures) named in the right-hand side of (39), would not be able to understand what it is to be a skyscraper. Similarly, if someone does not already know what fish, birds, amphibians, reptiles or mammals are, they would also hardly understand what 'vertebrates' means when they learn the enumerative definition in (40). Ostensive definition overcomes this problem by actually pointing at the object while giving the audience a definition of a term. Below is an example:

> Example (41): This is a pen (*while pointing at a pen*); this is a man (*while pointing at a man*). A pen and a man (*pointing at them respectively*), you know.
>
> Example (42): This is what I call beauty (*while pointing at the painting of* Mona Lisa).

From (41), it is easy to see that ostensive definition is a very primary and primitive way to introduce a linguistic expression. Proper names are often introduced that way, too. For example, a priest may point at a baby and declare 'this is John' and from then on the baby is called 'John'. Philosophers call this explanation of names a causal theory of naming.

Bertrand Russell (1912) regarded ostensive definition as privileged because it builds on knowledge by acquaintance which is direct and infallible in nature. Terms such as 'this' and 'that' are the only logically proper names because they alone indicate an ostensive definition taking place so the audience is presented with the object to be defined directly. Knowledge by acquaintance is contrasted with knowledge by description, in which people know of something just by learning that it satisfies certain descriptions. Knowledge by acquaintance is direct and certain according to Russell, whereas knowledge by description can be mistaken.

The above claims are convincing when concrete singular objects are involved. However, this method may not be effective as far as the definition of abstract concepts or a plurality of objects is concerned. Example (42) raises exactly such an issue. It is absolutely fine to define the painting of *Mona Lisa* by pointing at the painting itself; yet beauty is an abstract idea. Perhaps people may not unanimously agree that a particular object is beautiful. Perhaps beauty is found in a much wider variety of things. Perhaps it is even non-definable!

▶ 3 Recursive definition or inductive definition

Enumeration and ostensive definitions are non-exclusive. Recursive definition, sometimes called inductive definition, overcomes this problem. It does not overcome this problem by giving criteria or descriptions which objects must satisfy before they fall under a certain name. Rather, it does so by picking up objects systematically through a *repeated* procedure and forming a closure to ensure that nothing is left out. It is often used in defining mathematical and legal terms. Let us use some examples.

> Example (43): 'Positive integer' $=_{df}$ (a) 1 is a positive integer, indeed the smallest positive integer. (b) If n is a positive integer, $n + 1$ is a positive integer. (c) Nothing else apart from those fulfilling (a) or (b) is a positive integer.
>
> Example (44): 'A person is entitled to a holding' $=_{df}$ (a) A person who acquires a holding in accordance with the

principle of justice in acquisition is entitled to that holding. (b) A person who acquires a holding in accordance with the principle of justice in transfer, from someone else entitled to the holding, is entitled to the holding. (c) No one is entitled to a holding except by (repeated) applications of (a) and (b). (Robert Nozick (1974), *Anarchy, State and Utopia*, p.151)

(43) is a typical example of a recursive definition. It starts with (a) enumerating an example of an object falling under the name of the *definiendum*, which is a typical denotative definition procedure. Then it works its way in (b) to include all other numbers that fall under the same category by relating them to the first example. Finally, in (c) it gives the definition closure by stating that nothing else would fall under the category, making the combination of (a) and (b) not only the sufficient condition for anything falling under the category but also the necessary condition. By performing such three-step procedures, the definition succeeds in exhausting all positive integers, even though there are infinitely many of them. Indeed, most recursive definitions involve three components; they are called a base case (such as a), an inductive clause (such as b) and an external clause (such as (c)).

(44) works similarly. The (a) clause provides a base case in which a justified holding takes place such that an object is transferred justifiably from an unowned status to that of being owned by somebody. The (b) clause then specifies a further inductive operation: how an object transfers from being owned by one person to being owned by another. Finally, the (c) clause seals the deal by claiming that nothing other than repetition of (a) and (b) is justified. Observant readers will notice that instead of giving the audience sample cases that satisfy (a) or (b), the whole definition only specifies the criteria to be fulfilled. Hence, indeed, more information is needed to cash out items such as 'the principle of justice in acquisition' and 'the principle of justice in transfer', which is exactly what the author went on to give in his book.

One may wonder why recursive definition is considered a denotative definition because a recursive definition may contain connotative clauses as constituents (such as 43b, 44a and 44b).

Indeed, (44) does not even contain any enumeration or ostensive definition. This is a fair observation. Recursive definition is categorized under denotative definition because the overall aim of a recursive definition is to pick out the complete set of objects falling under the *definiendum*. How those objects are picked out (say, what principles of justice are involved in (44)) are really beside the point of the definition.

Another observation is that a recursive definition almost always looks like a circular definition. In (43), the *definiendum* ('positive integer') appears also in the *definientia* (43a and 43b). Similarly, in (44), the term 'entitled to a holding' repeats itself in the *definientia*. The reply is that at least the base case is not defined circularly. So the mentioning of the *definiendum in* the *definientia* does not give any substantial knowledge about objects in the world, but just circles around in the same conceptual level. (43a) specifies a particular object as a positive integer. (44a) specifies a basic principle by virtue of which a person entitled to a holding is picked out, yet the principle itself does not make use of the concept 'entitled to a holding'. Contrast them with the following example of a circular definition.

Example (45): A cause is whatever causes an effect.

It would not help to replace 'causes' with another expression, say, 'leads to' or 'results in'. For what defines an effect is still the outcome of a cause. Cause and effect, in this sense, are mutually defined.

To sum up, denotative definitions pick out objects or events in the world and so they link language to reality. They embody substantial knowledge about objects in the world. However, they are also typically non-exclusive because human beings usually do not have full knowledge about all objects in the world and many terms are meant to pick up an infinity of objects. Recursive definitions are exclusive; however, this kind of definition only applies to particular fields of inquiry.

A further disadvantage of the denotative definition is that it is not suitable for defining concepts which do not have a denotation, such as 'unicorns', 'mermaids', 'Vulcan' (as used

in astronomy to refer to a hypothetical planet which was later found to be non-existent). Explanation of empty names is indeed a notorious problem for philosophers of language who favour an externalist theory of names. The overall result is that denotative definitions may not always provide a clear and distinct boundary between objects falling under the *definiendum* and those not.

Connotative definitions, in contrast, take another approach by not listing objects themselves directly but rather specifying some properties or descriptions an object must satisfy in order to fall into the category of the *definiendum*. Three types are introduced below. The last one is the most commonly used type and we shall discuss more about rules to make a good definition of this kind.

▶ 4 Synonymous definition

Synonymous definition is simple. It defines something by providing a term with the same meaning (synonym). This is the kind of definition we typically find in dictionary entries.

Example (46): 'Enormous' $=_{df}$ 'huge', 'large', 'gigantic'

Example (47): 'Siblings' $=_{df}$ 'brothers or sisters'

Example (48): 'Amigo' (in Spanish) $=_{df}$ 'friends' (in English)

This method is easy, efficient and helpful. But it also has serious limitations. Many terms simply have no synonyms. There may be terms close in meaning, yet it would be wrong or confusing to replace one with the other. Moreover, this method only works if the *definiens* is already understood. However, sometimes a synonym would be as difficult to understand as the *definiendum* itself. For example, for a person who does not know the meaning of 'gigantic', explaining 'enormous' by 'gigantic' would not be helpful to that person at all.

There is also the problem of how to define synonymy itself. What counts for two expressions to mean the same? An intuitive answer would be that it is possible to *substitute* one with another without changing the meaning. Yet what counts

as a change in meaning? Does it mean only that there is no change in truth-value of the sentences constituting them? If this is the case, Hesperus and Phosphorus refer to the same heavenly body, Venus. So, say, every time Hesperus shines in a certain location, Phosphorus also shines there. However, it does not seem to make 'Hesperus' and 'Phosphorus' synonymous because intuitively, a person who *knows* 'Hesperus is Hesperus' may not know 'Hesperus is Phosphorus' even though these two sentences indeed share the same truth-value. The question is: should synonymy reflect cognitive status as well or just truth-values?

Take another example: suppose synonymy is defined as intersubstitutivity without changes in truth-values (the Latin term for intersubstitutivity is *salva veritate*). Empty names do not denote anything; hence, it is intuitive that they cannot make any sentence true. Suppose then all sentences containing empty names are not true sentences. Then substituting one empty name with another by default would cause no change in truth-values. Using the above-mentioned intersubstitutivity definition of synonymy, it would follow that all empty names are synonymous. If so, next time you read a Sherlock Holmes story, you can just replace 'Sherlock Holmes' with 'Watson' or even 'Moriarty' and the same meaning would be preserved. Does that sound right?

▶ 5 Operational definition

Operational definition defines a term by some *observational* attributes. It does not ask for any intrinsic properties or causes of observational attributes, because those properties, structures or causes may be complex and are exactly the subject matter to be studied. Yet in order to start the study, researchers need to delineate the scope of the study and hence they must adopt some criteria, no matter how superficial, as a boundary to constrain the scope. Operational definition fulfils this role. It is often used in science and social science disciplines.

> Example (49): 'Acid' $=_{df}$ the substance that turns a litmus paper red.

Example (50): 'People living in poverty' $=_{df}$ people who earn below half of the median household income of the society they live in.

Example (51): 'The sensation of pain' $=_{df}$ what causes behaviours of pain.

Operational definitions have to be 'operational'; that means, the *definiens* must involve some operation or procedure that is empirically testable and easy to spot practically. This implies that operational definitions are always constructed for some specific purpose. For example, academics may disagree on the definition of poverty as listed in (50), arguing that poverty is not only objective (measurable in monetary terms) but also subjective as reflected in social relationships and states of mind (including social exclusion and disadvantaged power status), or that it should be measured not only in terms of income but also expenditure. These are perfectly legitimate debates. However, (50) is established only for the purpose of counting and monitoring the number of poor people by certain groups of people, thus to provide suggestions for policy makers in distributing welfare resources. It is important to note that operational definition may not (perhaps should not) be the only definition offered for the *definiendum* concerned.

Similarly, we should also be aware that sometimes a certain operational definition already presupposes a certain way of theorizing the *definiendum*. For example, (51) is completely harmless as an operational definition for the study of pain. However, there is a theory in the philosophy of mind which takes the sensation of pain as *but nothing more* than the behaviours of pain. This is called behaviourism and it denies the existence of mental states. There are other different theories though. For example, identity theory does not deny the existence of mental states but identifies mental states as physical states. Functionalism reduces mental states to causal patterns between input (stimulus) and output (response and behaviour). Thus, if the same input–output pattern is observed, it would not matter whether this is called a mental process or a physical one. The point I want to illustrate here is that while operational definition is practical and effective, it cannot

replace theoretical discussions and it should not be taken for granted as the only appropriate definition. Doing so would presuppose a high theoretical commitment that operational definition is not meant for.

▸ 6 Definition by genus and difference

The last type of definition is probably the most widely available one. It works by identifying a larger group (genus) to which the *definiendum* belongs, then distinguishes its unique features or essential properties (difference) from other members of the genus. The definition works because most objects possess many attributes and some attributes are analysable into more attributes. By identifying the genus, this type of definition distinguishes the *definiendum* from a lot of other objects in the world. The latter step then fine-tunes the attributes to locate the essential features within the group. The relation between the larger group and the *definiendum* is sometimes described as that between a genus and a species, terms borrowed from biology though in this context they just carry relative meaning to indicate a class and a sub-class. Some examples are listed as follows.

Example (52): Human beings are rational animals.

Example (53): Hexagon $=_{df}$ polygon having six sides

In (52), 'animals' is the genus, 'human beings' is the species, 'rationality' is the difference that differentiates human beings from other animals. Similar structure is found in (53), where 'polygon' is the genus, 'hexagon' is the species, 'having six sides' is the difference.

The method of definition by genus and difference applies only to things that have complex attributes. However, there are arguably things that are simple and unanalysable and thus cannot be defined by this method, such as moral good, justice, and knowledge. To deal with these kinds of *definiendum*, we may appeal to other means, such as enumeration, operational definition, theoretical constructions, or simply leave them as they are to become primitive terms in theories.

Apart from the above limitation, there are also universal attributes that apply to everything, thus they are not useful if we use them to define a genus, for example: existence, substance, being, object, entity. Indeed, sometimes it is argued that existence is not an attribute at all because things have to exist before they can take on any attribute. Existence is thus a pre-condition of attributes, rather than being one of them. Famous philosophers who pursued this line of thought include Immanuel Kant and Gottlob Frege.

▶ Rules for definition by genus and difference

Since definition by genus and difference is used quite widely, there are some rules to help us appraise whether a definition of this kind is a good one. They are of course just rules of thumb but they serve well as tools for critical reasoning.

1 A definition should state the essential attributes of the species.

Definitions clarify meaning by giving clear and distinct boundaries to distinguish what things fall under a certain name and what do not. The clearer and the more distinct a boundary, the better. Hence, it is also best that, as far as possible, the definition by genus and difference identifies only attributes *uniquely* and *necessarily* possessed by the object in question. Sometimes scientists, logicians, philosophers, even religious people may believe that there is essence in each type of thing. What they mean by 'essence' may vary though. Let us illustrate the problem of not stating the essential attributes with the following examples.

> Example (54): A table is a piece of furniture with a flat top and one or more legs, providing a level surface for eating, writing, or working at.

> Example (55): A table is a piece of furniture usually rectangular and made of wood.

Which definition (54) or (55) captures better the essential attributes of tables? It is easy to see that (55) has shortcomings because, obviously, tables come in all shapes and materials and are not necessarily rectangular or made of wood. So, being

rectangular and made of wood are not the essential attributes of tables. In comparison, (54) is better as it captures the function of a table rather than just its outlook. However, we can still see that it is not perfect because there can be reclining tables specially designed for painting, graphic designing or blueprint drafting, etc. A table may have an uneven top to cater for a special need – perhaps the task to be done requires an uneven finish. An uneven top may also be designed for aesthetic or decorative reasons. In any case, (54) is probably good enough for common usage. We thus see what counts as 'essential attributes' is relative to time and context and the concept itself is perhaps one that is difficult to define.

2 A definition must be neither too broad nor too narrow.
tiêu chuẩn
This is a very useful criterion. A definition is too *broad* when the *definiens* includes more things than just the *definiendum*. A definition is too *narrow* if the *definiens* excludes something that should be part of the *definiendum*. The issue is simply a matter of scope. It may or may not relate to whether the attribute is necessary or unique to the *definiendum*. Hence, this rule is not the same as rule 1 (see above).

Example (56): Taxi $=_{\text{df}}$ terrestrial vehicle

Example (57): Human beings $=_{\text{df}}$ animals who understand the theory of relativity

Example (58): Human beings $=_{\text{df}}$ featherless bipeds with broad nails

The definition in (56) is too broad. Many vehicles run on land: cars, bus, trucks, vans… A taxi is just one type. The definition in (57) is too narrow. Indeed, not many human beings understand Einstein's theory of relativity. If (57) is accepted, then those who do not understand the theory of relativity will be excluded out of the category of humanity. (58) is the definition given by Plato. It is interesting because it is both too broad and too narrow. Can you see how? The definition is too broad because there are featherless bipeds that are not human. If this definition were good, then when we pluck all feathers out of a chicken or an ostrich and manicure its nails, the creature will thereby become

human, which is absurd. The definition is also too narrow because according to it, humans losing a leg, or having broken nails, for instance, would thereby stop being human just because they do not fit the definition any more.

3 A definition must not be circular.

dựa trên hiểu biết nhận thức

We have already touched on this point briefly in the discussion of recursive definition. A definition should not be circular because a definition is a cognitive tool to help people understand the *definiendum* more. A circular definition does not increase knowledge. It rather just uses the *definiendum* to formulate the *definiens*. Hence, for those who do not already know the meaning of the *definiendum*, the definition does not serve any purpose in enriching their knowledge or clarifying the meaning for them.

> Example (59): Economics $=_{df}$ the study of economic behaviour in human society
>
> Example (60): Well-being $=_{df}$ the state of being well

(59) and (60) are quite obvious examples. The word 'economic' appears in the *definiens* yet it it is exactly the term to be defined. 'Well-being is being well' also tells us nothing informative or new. However, sometimes circular definitions are not that obvious but may come in disguise. The same words may not be repeated, yet the *definiens* can only be explained in terms of the *definiendum*. In (45), 'cause' and 'effect' are indeed mutually defined even though apparently different words are used in the *definiens* and the *definiendum*.

4 A definition must not be expressed in ambiguous, obscure or figurative language.

Definitions are tools to clarify meaning. Therefore, it is expected that the *definiens* is easier to understand than the *definiendum*. A definition using ambiguous, obscure or figurative language is counter-productive and self-defeating to the purpose of definition. The following is an example because the *definiens* is even more obscure than the *definiendum* itself.

> Example (61): Life is the art of drawing sufficient conclusions from insufficient premises.

5 A definition should not be negative where it can be affirmative.

In general, it is more helpful to say what a *definiendum* is, rather than what it is not. Therefore, it is in general more useful to use an affirmative form instead of a negative one in a definition. However, some terms are essentially negative in meaning. So it would be alright to define them negatively.

> Example (62): A triangle is not a four-sided figure.
>
> Example (63): Orphan $=_{df}$ a child without parents
>
> Example (64): Absence $=_{df}$ something missing, not being present

(62) is not a good definition because there are many figures that are not four-sided and yet not a triangle. However, an orphan is essentially someone who has lost their parents. So it is impossible to define 'orphan' without saying what is absent. The same case goes with (64).

To sum up, in this section we have discussed six types of definition: three belonging to denotative definitions and three to connotative definitions. The denotative definitions are: enumeration, ostensive definition and recursive definition. The connotative definitions are: synonymy, operational definition, and definition by genus and difference. We also introduced five rules to evaluate definitions by genus and difference. They are:

1 Specify essential attributes

2 Be not too broad nor too narrow

3 Avoid circularity

4 Avoid ambiguous, obscure or figurative language

5 Be affirmative

We can now employ these rules in an exercise.

Exercise 2.2: Evaluating definitions

Criticize the following in terms of the rules for definition by genus and difference. Identify the rules violated and explain why.

1. An oil painting is a picture drawn on a canvas with a brush.
2. Faith is the substance of things hoped for, the evidence of things not seen. (The Bible, Hebrews, 11:1)
3. Well-being is a positive outcome that is meaningful for people and for many sectors of society, because it tells us that people perceive that their lives are going well.
4. The more specific idea of Evolution now reached is – a change from an indefinite, incoherent homogeneity to a definite, coherent heterogeneity, accompanying the dissipation of motion and integration of matter. (Herbert Spencer (1898), *The Principles of Sociology*, Chapter 16, §129)
5. Love is nature's way of tricking people into reproducing. (Anonymous)
6. 'The meaning of a word is what is explained by the explanation of the meaning.' i.e.: if you want to understand the use of the word 'meaning', look for what are called 'explanations of meaning'. (Ludwig Wittgenstein (1958), *Philosophical Investigations*, Section 560)
7. Poetry is lofty thought or impassioned feeling expressed in imaginative words. (http://www.dictionary.com)
8. Religion, noun. A daughter of Hope and Fear, explaining to Ignorance the nature of the Unknowable. (Ambrose Bierce, *The Unabridged Devil's Dictionary*)
9. Science is systematic knowledge of the physical or material world gained through observation and experimentation.
10. Justice is the quality of being just.

2.5 Chapter summary

We started this chapter by noting the formal character of deductive logic. A deductive argument is valid or invalid irrespective of its content, but simply dependent on its argument form. Having said that, meaning does play an important role in reasoning because often we cannot even

formulate and formalize an argument correctly if its meaning is not clear or we have misunderstood it. This drove our discussions in the rest of this chapter. Attention was directed particularly to ways to clarify meanings.

Several types of linguistic phenomena related to unclear expressions were identified first. They included: ambiguity, vagueness, obscurity, incomplete expressions, idiosyncrasy, and the use of emotive words. An expression is ambiguous if it has more than one meaning. It is vague if the concept has no clear boundary and the judgement is a matter of degree. An expression is obscure if it even lacks any core meaning. Incomplete expressions are those that may imply hidden contextual restrictions not yet made explicit. Idiosyncrasy refers to novel meanings made up for the discourse not yet clearly spelled out. Finally, emotive words may elicit psychological or rhetoric effects without arguing explicitly why such effects are justified. All these problems are to be avoided as far as possible for clear thinking.

It seems unfair to discuss just what meaning is not or what problems should be avoided without eventually discussing what it is positively. Section 2.3 performed that task. However, this section also includes reference to important philosophical discussion. One significant point is that we have to recognize that by 'meaning' we often mean a lot of things. Sometimes we mean it as reference; to clarify the meaning of a word is to find out the object to which it refers and to clarify the meaning of a statement is to determine whether it is true. However, there is more to it. Meaning sometimes means something in virtue of which reference is picked up. Such a thing is called variably sense, intension, or connotation.

Another important distinction are analytic and synthetic judgements. Analytic statements are often conceived as being necessary and a *priori*; whereas synthetic statements are contingent and a *posteriori*.

The final section focused on definition, an important tool to clarify meaning. Several types of definition were discussed.

In particular, we introduced some rules for evaluating definitions by genus and difference.

In the next chapter the focus is still on meaning. We shall discuss fallacies related to the misuse of language and other informal matters.

3

Informal fallacies

ý kiến sai lầm.

In this chapter you will learn about:

- ***fallacies from the abuse of language***
- ***fallacies of relevance***
  - ***fallacies of appeal***
  - ***fallacies about premises and conclusion***
  - ***fallacious induction***

'It's very good jam,' said the Queen.

'Well, I don't want any to-day, *at any rate.' (Alice)*

'You couldn't have it if you did *want it,' the Queen said. 'The rule is, jam to-morrow and jam yesterday – but never jam to-day.'*

'It must *come sometimes to "jam to-day,"' Alice objected.*

'No, it can't,' said the Queen. 'It's jam every other *day: to-day isn't any* other *day, you know.'*

Lewis Carroll (1871), *Through the Looking Glass*, Chapter 5

We discussed in Section 2.2 some types of unclear linguistic expressions. Not only does the abuse of language lead to misunderstanding, it can also lead to fallacious reasoning. A **fallacy** is a type of argument that may seem to be correct but on examination is not so. The fallacies due to misuse of language or irrelevance are usually called **informal fallacies,** to distinguish them from formal, logical fallacies that form invalid arguments (to be introduced later in Chapters 4–6). There are two main types of informal fallacies: fallacies arising from the abuse of language and fallacies of relevance. Just as there are different types of unclear expressions, there are also different types of fallacies arising from the abuse of language.

3.1 Fallacies from the abuse of language

EQUIVOCATION

nói hai nghĩa, lập lờ.

The fallacy of **equivocation** occurs when the same word or phrase is used with two or more meanings, deliberately or accidentally, in the formulation of an argument. This is a fallacy due to ambiguity or vagueness.

> Example (1): Time is money. Time heals all wounds. Therefore, money heals all wounds.

The word 'time' is ambiguous in this example and the speaker employs this ambiguity to make the argument. The argument even looks valid: if 'is' means identity and we can substitute identical

thay thế

things with each other, then substituting 'time' with 'money' in the second sentence give us the words as they appear in the conclusion. However, it is in fact not a valid argument. The reason is that the word 'time' has different meanings here, so we just can't simply replace one token of it with what is predicated of another token.

xác nhận, khẳng định

In the first sentence, when we say that time is money, we mean making money in a timely manner, namely to be efficient and opportunistic. Action and efficiency are implicated. However, in the second sentence, the exact opposite is implied. It suggests patience, calming down, letting the pain pass and waiting for the body and mind to recover. The point here is the passage of time. It would have nothing to do with earning a fortune and certainly money cannot buy you the time if you have none, or spare you from undergoing the healing process at all. Hence, although the same word 'time' is used, the implications are actually very different. Time is not a thing but a condition under which things can happen. One token of the word may not be replaced by another. Rather, we need to investigate whether the terms are used in the same sense.

hàm ý, ám chỉ

If this example is not clear enough, let's look at another one:

> Example (2): Esther likes to stroll along banks. A bank is a financial institution. Therefore, Esther likes to stroll along financial institutions.

The argument is fallacious because the word 'bank' has different meanings in its two occurrences. In the first occurrence, it means embankment; in the second it means a place where people deposit and withdraw their money. Although the same word is used, it does not mean they are interchangeable.

The above two examples are obvious and do not seem harmful. It's doubtful whether anyone would take the argument in them seriously. There could however be serious mistakes made from arguments involving equivocation, particularly in philosophical discussions, where accuracy in handling of concepts is of particular significance.

> Example (3): 'To have an attitude of respect for nature is to regard the wild plants and animals of the Earth's natural ecosystems as possessing inherent worth...

[A teleological centre of life has inherent worth]... To say [an organism] is a teleological centre of life is to say that its internal functioning as well as its external activities are all goal-oriented, having the constant tendency to maintain the organism's existence through time... We need not, for example, consider them to have consciousness. That a particular tree is a teleological centre of life does not entail that it is intentionally aiming at preserving its existence...' (From Paul Taylor (1986), *Respect for Nature*, Princeton University Press)

This passage argues that both plants and animals 'possess inherent worth' because they are 'teleological centres of life'. Thus, both plants and animals should enjoy the same moral status and be respected. We can simplify and reformulate the argument as follows.

Wild plants and animals are teleological centres of life.

Anything that is a teleological centre of life has inherent worth and should be respected.

Therefore, wild plants and animals have inherent worth and should be respected.

However, why are wild plants and animals teleological centres of life? What does it mean to be a teleological centre of life? The passage defines a teleological centre of life as 'goal-oriented'. There are two possible interpretations of this term:

1 **It is in X's interest that X survives.** There is a constant tendency for X to maintain existence, observed through an external point of view. This is the objective sense of interest or value.

2 **X is interested in surviving.** The organism intentionally aims (sets its goal) at preserving its own existence. This is the subjective sense of interest; only conscious beings can do this. This is also the common sense understanding of goal-orientation.

It is under the first sense that the author argues for the same moral status of non-conscious organisms as sentient humans

and animals. However, normally we ascribe moral duties to someone only if they possess a goal of their own in the second sense. For example, it is morally wrong to harm human beings because they feel pain and human beings in general *intentionally* seek pleasure and avoid pain. If an object does not seek pleasure or avoid pain at all (like a human in a vegetative state or a robot), it is arguable whether it is wrong to inflict pain on that object. Hence, the second sense of interest may not be sufficient for moral status. The author needs to provide more arguments for this and cannot simply shift from arguing that it is in the interest of non-sentient beings that they survive to the conclusion that they are interested in survival.

AMPHIBOLY

có vẻ hợp lý
miếng lưỡi khéo léo

Amphiboly occurs when one of the statements in an argument has more than one plausible meaning because of the loose or awkward way in which the words in that statement are combined. **Equivocation** is due to multiple meanings of a phrase or a word; it is semantic in nature. However, amphiboly is caused by syntactic ambiguity, namely, alternative ways of taking the grammar.

∈ ngữ nghĩa

cú pháp

> Example (4): see the quoted passage at the start of this chapter, from *Through the Looking Glass* by Lewis Carroll.

The word 'to-day' is ambiguous here. When Alice says it, 'to-day' simply refers to the same day when the utterance is made. Thus, suppose I said on 5 February 2016 that 'it is cold today', and I report it on 6 February 2016, I should say that it was cold yesterday. Semantic rules demand that we make such adjustments in speaking and interpretation. Context-sensitive phrases like this and many others are categorized as indexicals. Apart from temporal indicators (*now*, *then*, *tomorrow*), other examples include pronouns (*I*, *you*, *he*), and location words (*here*, *there*), etc.

The Queen however did not seem to follow the same grammatical rule here. She took 'jam to-day' as a fixed term like a proper name (like 'the Robertson jam') or a general name (such as 'raspberry jam') and went on to compare it with 'jam other-day' word by word to locate their difference.

This is how she arrived at the conclusion that 'jam to-day' never comes. Her (implicit) argument can be reformulated as follows:

> [Jam comes every other day. So what comes is jam-every-other-day.]
>
> Only jam-every-other-day comes.
>
> Jam-to-day is not jam-every-other-day.
>
> Therefore, jam-to-day does not come.

This type of ambiguity arises out of the misapplication of grammar. There are two readings of 'jam' in this example: *jam to-day* (as normally used in indexical form, and understood by Alice) and *jam-to-day* (as a proper name in itself, and understood by the Queen).

> Example (5): The King of Lydia consulted the Oracle of Delphi before beginning his war with the Kingdom of Persia. The Oracle replied, 'If the King of Lydia went to war with Cyrus, he would destroy a mighty kingdom.'

Upon hearing the Oracle, The King of Lydia thought that it meant he would destroy his enemy. So he started the war and lost. Do you think the Oracle has wronged him? The Oracle can be interpreted in two ways. 'He would destroy a mighty kingdom' can be taken as to answer, in the context of war, what would happen to the opposing side in the war. Or, it may mean as a question about the welfare of the country that starts the war. The first reading yields the interpretation that the King would destroy the Kingdom of Persia. However, the second reading generates the understanding that the King would destroy the Kingdom of Lydia. No matter whether the King of Lydia won or lost, the Oracle was not mistaken in either way.

This is amphiboly because the ambiguity is caused by two ways of reading into what is being consulted: the future of the opponent or the war-wager. This is not a case of equivocation because although 'mighty' is a vague term, the ambiguity is not caused by this feature. Rather, the ambiguity is inherent in the indefinite article 'a' mighty kingdom, which allows different interpretations as to which one.

ACCENT

Accent is a shift in meaning arising as a consequence of changes in the emphasis given to its words or parts, thus leading to fallacious reasoning.

Example (6):

A: We should not speak ill of our friends. So why do you speak *ill* of John?

B: Because we should not speak ill of *our friends*!

[Implicit meaning: 'John is not our friend'.]

The change in the emphasis of the words in this conversation represents the two arguments A and B put forward. Note how opposite conclusions are drawn.

A:We should not speak ill of our friends.

<u>John is our friend.</u>

Therefore, we should not speak ill of John.

B:We should not speak ill of our friends.

<u>John is not our friend.</u>

Therefore, we should speak ill of John. (Or, it is alright to speak ill of him.)

B's argument is not fallacious in itself. However, if A's argument is what is meant, then by changing the accent, B has at least twisted or misled the original argument in another direction. It is in this sense that we say this example commits a fallacy.

SLIPPERY SLOPE ARGUMENT

In a **slippery slope** argument, a small concession is presented as having potentially catastrophic consequences. This type of argument exploits vagueness erroneously to the reasoner's advantage.

Example (7): It's shocking what people wear these days. The other day I saw a young lady with her ankles clearly exposed. Today I saw someone wearing a miniskirt. If things carry on at this rate, we'll soon all be walking around naked.

Ordinary matters often come in degrees. Many issues are benign when on a small scale. However, a slippery slope argument uses this to argue for an alarming result. It is a fallacy because in fact, things may not be as bad as is claimed, and there are usually many intermediary stages which will stop a situation from developing to the extreme. Thus, it is not the case that a catastrophic consequence must follow in the way the argument wants to convince its audience that it will.

Take the above example. From exposing an ankle to walking around naked, there are indeed many steps, and so many conditions and factors to take into account. Hence it is fallacious to argue that from one end of the story, the other end must follow.

Slippery slope arguments are quite commonly found in daily life. Typical examples are found in government red tape or bureaucracy.

> Example (8): A government officer receives a request from citizens and answers, 'Oh, we cannot entertain this request. For if everyone makes the same request, the whole system will collapse.'

This is a fallacious argument for two reasons. (1) Not everyone would make the same request. Indeed, the officer may have no evidence to support the claim that everyone would make this request.(2) More importantly, if everyone does make the same request, it is the government that needs to change its set-up (budget, manpower, schedules, etc.) to meet a request that everyone demands, rather than to avoid and deny the request from the start! This is what a government, at least an accountable government, is meant to do.

It is important that when we accuse arguments of committing fallacies, formal or informal, we explain *why it is fallacious* in that particular case. Arguments deploying vagueness are not automatically fallacious. Critics need to argue why the deployment of vagueness in that argument is mistaken. For instance, although Robert Nozick (1974) employed vagueness to argue against taxation as on a par with forced labour and

slavery (see my paraphrased text below), it is arguable whether the argument commits the slippery slope problem.

> Example (9): Surely you are a slave if your life is completely at the mercy of your brutal master who often beats you cruelly and calls you out in the middle of the night. If your master is kindlier and beats you less often, even makes rules to govern your work, it does not change the fact that you are his slave. Suppose your master even let you work outside but you must hand in all your earnings, you no doubt are still a slave. Now if your master let you hand in only part of your earnings, do you thereby become not a slave? Taxation is coercion to hand the fruits of your labour. Therefore, taxation is forced labour and slavery. (Adapted from Robert Nozick (1974), *Anarchy, State and Utopia*, Oxford: Blackwell, 290–292)

The argument is that since there is not a clear line at which the slave ceases to be a slave, the slave remains a slave all the time. This argument certainly appeals to the vagueness of the concept of 'slavery'. Yet whether this is a fallacious argument has to be debated and cannot be accused just because of the form of the argument.

FALSE DICHOTOMY/FALSE DILEMMA

An argument commits the fallacy of **false dichotomy** or **false dilemma** if it presents two alternatives as exhaustive, when in fact other possibilities exist.

> Example (10): If you eat too much, you'll be stuffed to death. If you eat too little, you'll starve to death. You eat or you don't eat. Hence, no matter how much you eat, you'll die.

> Example (11): To be rich, I've got to inherit money or marry well. I'm not going to get a penny from my parents, so I'd better marry well.

> Example (12): Either we must support the bourgeois ideology or the socialist ideology. We must not support the bourgeois ideology. Therefore, we must support the socialist ideology.

A dilemma is a valid form of argument. It has many forms but it typically argues in the following manner: first it establishes two options (called the two horns), each leading to some particular consequence, then it goes on to evaluate these consequences. If the two options are the only available options, it follows that one of the consequences must be the case. The example below is a proper dilemma argument.

> Example (13): If I take the job, I sacrifice my family time so my family suffers. If I don't take the job, money will be tight and my family will suffer, too. Either I take the job or I don't. Hence, my family suffers anyway.

Example (13) is valid and is indeed the struggle of many working mothers. However, there may be ways to get out of this – for example, the degree of the seriousness of suffering may differ in different options and we may still be able to choose the lesser evil. We may even reconstruct the dilemma as follows to make us feel better!

> Example (13a): If I take the job, I earn more money and my family benefits. If I don't take the job, I have more family time and my family benefits. I either take the job or I don't. Hence, my family benefits anyway.

The cases in examples (10)–(12) are different, though. Those examples all have false premises, namely the two horns are not the only exhaustive options.

In (10), death happens only when you eat extremely little (starved to death) or excessively to the extreme (stuffed to death). This does not imply you must either eat extremely little or excessive. Indeed, there are a lot of in-between cases where people eat moderately and stay healthy.

Similarly, (11) is a false dilemma because inheritance and marrying well are not the only ways to get rich. Thus, the dichotomy offered in the first premise is false. In (12) the bourgeois and the socialist ideologies are not the only ideologies present in the real world. There is welfare liberalism, libertarianism, communitarianism, socialism of various types, even philanthropy, to name but a few. It is thus false to assume that one can be either a bourgeois or a socialist.

STRAW MAN FALLACY

The **straw man fallacy** occurs when we level our arguments at a crude caricature of our opponents' views. This means we are just knocking down the wrong enemy (a 'straw man') that we have set up, rather than tackling the real, original issue. This move may steer people's attention towards the wrong issue and win the reasoner a rhetorical advantage. However, it really is bad reasoning because it responds to the wrong question and leaves the actual issue unresolved.

Example (14):

A: We should not discriminate against homosexuals.

B: So, are you saying that we should all be gay?

In (14), the original issue A raises is the discrimination of homosexual people. To give a proper response, B should be commenting on the same issue. Yet B does not do so. B twists the issue and turns it into whether we should all be homosexual. This is a different issue, for no matter whether one is or is not homosexual, or whether one approves or disapproves of homosexual behaviour, one may still think it is unjust to treat homosexual people differently. Discrimination is wrong because homosexual people are free and equal citizens like anyone else. Hence, if other people have natural rights as persons and civic rights as citizens, homosexual people should also have equal treatment. There is no reason to strip away their rights and treat them differently from other people and citizens. B sets up a false target, and an irrelevant proposition indeed, as if it were the original issue. This is obviously a fallacy. In setting up a straw man, B may gain some advantage because it is easy to arouse an emotional response in an audience that is against homosexual behaviour; yet it masks the possibility that heterosexuals would oppose discrimination against homosexuals.

RED HERRING

A **red herring** changes the subject and diverts attention to other issues.

Example (15):

Reporter: How would the Cabinet respond to the question of MPs claiming allowances for private entertainment?

Cabinet spokesman: There are more pressing issues at the moment; shall we talk about the budget cuts in education...

A red herring differs from the straw man fallacy in that the speaker does not assert or pretend to assert that the issue is the original issue or a proper development of the same discussion. For example, using the example of homosexual discrimination, they would not assert that the approval of gay behaviour is the real or the key issue to homosexual discrimination. They simply draw the audience's attention away from the original issue. They may not even deny such an intention if asked.

Strictly speaking a red herring is not a fallacy, for it does not aim at saying something false. Yet it is an improper move in reasoning because it turns away from the problem in question. It can however be a common strategy used in argumentation when someone just wants to avoid the issue.

COMPOSITION

The fallacy of **composition** is made when (a) one reasons mistakenly from the attributes of a part to the attributes of the whole, or (b) one reasons mistakenly from the attributes of an individual member of some collection to the attributes of the totality of that collection.

> Example (16): Every footballer in the All-Star team is a good player. Therefore, the team they formed is a good team.

In the fallacy of composition, a reasoner erroneously argues for properties of the whole from those of its components. It is in general not a problem to argue from whole to parts, or vice versa, from parts to whole. **Inductive syllogism** does the former; **inductive generalization** does the latter (see the box below). These are accepted forms of induction. However, sometimes the reasoning goes wrong because some properties do not *mean* the same when attributed to an individual and to a collective. For example, they may not be subject to the same kind of criteria or standard of assessment, or they may be subject to a different class of objects for comparison. So the fallacy of composition is related to abuse of language.

Spotlight: Examples of some forms of induction

Inductive syllogism: (from general pattern to individual)

Most college students are against tuition fee rises.

Mary is a college student.

Therefore, Mary is against tuition fee rises.

Inductive generalization: (from individual to general pattern)

All observed swans are white.

Therefore, all swans are white.

Take (16) as an illustration. The criteria for assessing a good individual football player are different from that for assessing a good football team. It is possible that each individual player is highly skilled in football playing – in kicking, passing the ball, shooting, etc. However, they may still not form a good team because they may lack coordination, team spirit, cooperation, or strategy. These are qualities that do not attribute to individual players alone. In other words, although the same word 'good' is used, a good player and a good team mean different things and are assessed according to different criteria and standards. Therefore, it is fallacious to infer from individuals being good players that the team they form is a good team.

DIVISION

The fallacy of **division** is formed when (a) one reasons mistakenly from the attributes of a whole to the attributes of one of its parts, and (b) one reasons mistakenly from the attributes of a totality of some collection of entities to the attributes of the individual entities themselves.

The fallacy of composition and the fallacy of division are a pair. They are fallacious because criteria applying to individuals may not apply to the whole population, and vice versa. The fallacy of composition argues from the individual to the population; the fallacy of division argues from the population to the individual.

> Example (17): The USA is an affluent country. Therefore, there are no poor people living in the USA.

The measurement for an affluent country is quite different from that for individual citizens. While the latter includes only household income and expenditure, the former may include GDP (Gross Domestic Product), average household income, etc., and measures the overall profit made by the country. In any case, being an affluent country does not imply that income and wealth are equally distributed among its citizens. The USA is well known for economic inequality. Its wealth gap is constantly widening, and has done so particularly in the past two decades. There are surely poor people in the USA. The inference in example (17) is fallacious because it mistakenly reasons from the attributes of the whole to those of its parts.

To sum up, in this section we have introduced nine fallacies that are all related to the abuse of language. Each is concerned with a different problem area of linguistic usage.

Key idea: Fallacies from the abuse of language

Equivocation: multiple meanings, semantic ambiguity

Amphiboly: multiple meanings, ambiguous grammar

Accent: multiple meanings, pragmatic ambiguity

Slippery slope: vagueness

False dichotomy/false dilemma: vagueness

Straw man fallacy: wrong understanding of the subject matter

Red herring: avoid discussion by turning to other issues

Composition: mistakenly arguing from parts to whole because of the presence of different criteria applicable to them

Division: mistakenly arguing from whole to parts because of the presence of different criteria applicable to them

Let us now test our understanding by doing an exercise.

Exercise 3.1: Informal fallacies (1)

Identify the fallacies, if any, in the following sentences and explain:

1 Once you legalize abortion, you inevitably end up with child pornography, child abuse, and abuse of the aged. Complete lack of respect for life breeds complete lack of respect for the living. Therefore, we must outlaw abortion.
2 If you think drug-taking is bad, you must support the war on drugs.
3 The only proof capable of being given that an object is visible is that people actually see it. The only proof that a sound is audible is that people hear it: and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people actually desire it. (John Stuart Mill (1861), *Utilitarianism*, Chapter 4)
4 If the parts of the Universe are not accidental, how can the whole Universe be considered the result of chance? Therefore, the existence of the Universe is not due to chance. (Moses Maimonides, *The Guide for the Perplexed*)
5 We may choose to die in honour or die in vain. No one wants to die in vain. Therefore, let's die in honour (and serve the army)!
6 An elephant is an animal. Therefore, a small elephant is a small animal.
7 A country is composed of people. What is good for the country must therefore be good for its people.
8 One grain of rice does not make a heap. If one grain does not make a heap, adding another grain does not a heap. We can only add one grain at a time. Therefore, there is never a heap.

3.2 Fallacies of relevance

Apart from fallacies due to misuse of language, fallacy also arises when an argument relies on premises not relevant to its conclusion and therefore cannot establish its truth. We call these types of fallacies in general **fallacies of relevance**. They are

fallacious because the relations they appeal to are psychological rather than logical. These fallacies are informal because relevance is often determined by content, not just by argument form.

Fallacies of relevance can be classified into several categories. We introduce three in this book. First, **fallacies of appeal** appear when an argument appeals to the wrong type of reason. **Fallacies about premises and conclusion** focus on the argument form, such as its going in circles or having inappropriate assumptions. Finally, **fallacious induction** concerns inductive arguments.

FALLACIES OF APPEAL

▶ Appeal to ignorance (argument *ad ignorantiam*)

The **appeal to ignorance** is a fallacy in which a proposition is argued to be true simply on the basis that it has not been proved false, or false simply because it has not been proved true.

> Example (18):
>
> A: God exists!
>
> B: Why?
>
> A: Because if God does not exist, then someone will know. But no one disproves that God exists. So God exists.

This is an argument of appealing to ignorance because it claims the conclusion is true just on the grounds that it has not been proved to be false.

> Example (19):
>
> A: God does not exist.
>
> B: Why?
>
> A: Because if he exists, then we would know. But we do not know he exists. So he does not exist.

This is also an appeal to ignorance because it claims something is false just when it has not been proved to be true.

Appeals to ignorance confuse fact and knowledge. It is one thing that a state of affairs is a fact (i.e. a proposition is true), which is a metaphysical matter. It is quite another whether it

is *known* that a state of affairs is a fact, which is a matter of knowledge or epistemology. We often think something is true when it is actually false. For example, people used to think whales are fish, when they are actually mammals. We also may not know something is true when it is indeed true. For example, before 1750 scientists had not discovered that water is H_2O. Yet water is H_2O. Whether we know it or not does not change the fact that it is true or false. Imagine we lived before 1750, then we were not able to prove that water is H_2O. Yet it does not mean that water is not H_2O. So we cannot reason that if we have not proved something to be true that it is not true.

Nor can we reason that if we have not proved something to be false to that it is true. We may give things the benefit of the doubt and suspend our judgement: perhaps it is true, perhaps false. Yet this is not the same as claiming it is true or false just because we cannot yet prove it to be otherwise.

In general, philosophers believe the following: things are known to be true only if they are true. However, that something is *not* known to be true implies neither that it is true nor that it is false.

> Knowing $p \rightarrow p$
>
> But: Not knowing p does not imply p or not-p.
>
> Proved $p \rightarrow p$
>
> But: Not proved p does not imply p or not-p.

Consider some more everyday examples.

> Example (20): There was no proof that Saddam Hussein didn't have weapons of mass destruction. Therefore, he had them.
>
> Example (21): There was no proof that you didn't cheat on me. Therefore, you did.

Imagine your partner said (21) to you. How would you feel? Furious, obviously, and with good reason too because the argument is obviously not a good argument! Yet (21) shares the exact argument form with (18) and (20). So, if (21) is bad argument, so are (18) and (20). The actual content of an argument may cloud our judgement of its validity. We may

not be able to immediately identify (18) and (20) as fallacious simply because we are not already sure whether God exists or Saddam Hussein had weapons of mass destruction. Yet the uncertainty is merely psychological and not logical.

Now we have explained appeals to ignorance and why they are fallacious, let us dive into more complicated cases and diagnose whether they are appeals to ignorance. Consider the following case.

> Example (22): The new drug has no toxic effect on rodents. The law forbids testing drugs on humans. So, there is no evidence that the drug will have a toxic effect on humans. Therefore, we can conclude that the new drug has no toxic effect on humans.

We cannot conclude that the new drug has no toxic effect on humans simply because there is no proof that it has any toxic effect on humans. Doing so would commit to the fallacy of appeal to ignorance. However, the case is not so simple because although we do not have evidence of the drug effect on humans, we do have evidence of its effect on rodents. Given that rodents and humans are similar biologically, it is not unreasonable to argue that similar effects on rodents might appear if the drug were to apply to humans (counterfactually). How strong the rodent cases bear on the case of humans depends on how biologically similar rodents and humans are. In fact, given that it is unethical to test on humans, the indirect evidence from the rodents may be among the few resources we can count on.

The above does not imply support of animal testing, nor that animal testing is the only source of knowledge in testing medicines; in fact I am opposed to animal testing. I am simply saying that it is not unreasonable to consider indirect evidence *if* no better resources are available. There are cases where drugs tested on animals are fine yet they still have no effect, or even cause harm, when applied to humans. A well-known example is thalidomide, used in the 1950s as a sedative and prescribed to treat morning sickness in pregnant women. Thalidomide was safe when tested in animals, but it led to birth defects in over 10,000 children when used by mothers-to-be.[1]

Similarly, caution has to be exercised in jurisprudence. In the common law system, defendants are presumed innocent until they are proven guilty. Other legal systems presume otherwise and take defendants to be guilty until proven innocent. In either case, can we say the court has committed the fallacy of appeal to ignorance?

On the face of it, the charge seems appealing. According to our common sense, the following metaphysical claim is true.

1 One is innocent (guilty) if and only if one has not (has) committed a criminal act.

So, if the court judges that a person is innocent, then the court admits that the person does not commit a criminal act. However, when the court judges that a person is innocent, it only deals with the following:

2 One is proven innocent (guilty) if and only if there is sufficient evidence to show that one has not (has) committed a criminal act.

The second example focuses on the epistemological aspects whereas the first example is a metaphysical claim. If the court judges innocence by proven innocence, then it may seem like that the court has confused metaphysics with epistemology. We can lay out the reasoning in the following manner:

1 One is innocent (guilty) if and only if one has not (has) committed a criminal act.

2 One is proven innocent (guilty) if and only if there is sufficient evidence to show that one has not (has) committed a criminal act.

3 If one is proven innocent, then one is innocent. And, if one is not proven innocent, then one is guilty.

4 Therefore, if one is proven innocent then one has not committed a criminal act. And, if one is not proven innocent, then one has committed a criminal act.

However, one may indeed have committed a crime without being known to do so or leaving any evidence behind. So,

the epistemological status of a case should not imply the metaphysical fact that the same person did or did not actually commit a crime.

The legal system however does not claim that (2) implies (1). When a defendant is professed innocent, it simply means legally that the defendant is proven innocent. It does not imply that the defendant did not commit the relevant *act*. It simply means either that the act is not *considered criminal* in legal terms, or that there is not enough evidence showing that the defendant has committed it. In other words, the court holds (2) all the way through and would not claim (1), (3) or (4).

In sum, the court does not commit to the fallacy of appeal to ignorance because it does not seek to establish the metaphysical from the epistemological. Terms like 'innocence' or 'guilt' are always employed in the courtroom with a special sense, which restrains them from drawing metaphysical conclusions as common sense suggests they would.

▶ Appeal to inappropriate authority (argument *ad verecunidam*)

When an argument appeals to some party having no legitimate claim to authority in the matter at hand, it commits to the fallacy of **appeal to inappropriate authority**.

The keyword here is 'inappropriate'. It is in general permissible to appeal to authority in making arguments. The contemporary world is so complex and knowledge is so diversified and specialized that hardly anyone can be expert in everything, if at all. Experts have the necessary experience, training and knowledge about their subject; hence they are justified in making informed judgements and having more say about the subject matter they know. However, being an expert in one area does not imply being an expert in another as well. Hence, if one appeals to the wrong authority, one simply loses the superior claim in argument because of that.

> Example (23): We are not morally wrong in bullying the weak. Hasn't Darwin shown that evolution relies on the survival of the fittest? It is the weaklings' own problem for not being able to survive.

Charles Darwin (1809–1882) is a biologist famous for proposing the Theory of Evolution. However, the Theory of Evolution does not automatically generate any specific implications on ethics and morality. Certainly there might be connections and some people would look at ethics from an evolutionary perspective. It remains true that at least Darwin would not claim himself as an expert in ethics. Hence, appealing to him as the authority in moral matters is inappropriate and irrelevant.

We may reconstruct example (23) as the following:

1 According to Darwin, evolution relies on the survival of the fittest.

2 The weak are not fit for survival.

3 Therefore, the weak won't survive in evolution.

4 Therefore, the weak should be bullied.

The argument from (1) and (2) to (3) is fine, though the term 'weak' is vague and ambiguous. At least it is not wrong to appeal to Darwin for explanation of changes in species. The problem, however, is the argument from (3) to (4). (3) is a *factual* proposition of whether the weak, however defined, survive or not. (4), however, is a *normative* proposition of whether an individual 'should' bully the weak. The former is a matter of 'is', whereas the latter is a matter of 'ought'. One ought not to do something (like hurting others for fun) even though one feels like doing it or has done it. Similarly, one ought to do something (like donate to charity or refrain from addiction) even when one does not want to. So the two realms are quite distinct. Reasons supporting (3) may not apply to (4).

It is easy to find daily examples of appeal to authority in advertisements. For example, toothpaste promoters may dress in medical or dental gowns even though they are not surgeons or dentists.[2] Sometimes advertisements may appeal to *inappropriate* authority too. For example, celebrities in one area may be invited to endorse or do commercials for products not related to their fields. Thus, chef Gordon Ramsay may feature

in the commercials for Gordon's gin, even though cooking and spirit drinking may not be directly correlated.[3]

APPEAL TO COMMON PRACTICE

Appeal to common practice occurs when an argument appeals to the fact that a view is universally or commonly held as support to its truth. It is fallacious because a view can indeed be commonly held yet false nonetheless. The appeal may trigger some psychological effects. However, it is not directly relevant to the truth of related propositions.

> Example (24): How can you be in favour of slavery? All progressive minded people think it is wrong.

(24) is worded as a question, yet the underlying argument can be reformulated as follows.

1 All progressive minded people think slavery is wrong.

3 Therefore, slavery is wrong.

(1) does not lead to (3) unless a further premise is added. This missing premise is:

2 If all progressive minded people think X, then X.

However, (2) is problematic and is likely to be false. What is wrong with this argument is not whether slavery itself is wrong but whether this conclusion comes from (1) and (2). It may well be that the conclusion is indeed true, but the way to argue for it by appealing to the populace is misguided.

> Example (25): Everybody said that Gordon Brown was going to resign, therefore he was going to resign.

Arguments like (25) can hardly be accepted for several reasons. (a) The word 'everybody' in this type of argument can hardly mean *everybody* literally – there were many people in the world who did not even know Gordon Brown and so would not say anything about him. That is to say, the domain of the word 'everybody' is likely to be restricted to a particular conversational context. (b) Even with a restricted context, in practice the word 'everyone' is likely to be used as a rhetorical device to emphasize that many (rather than all) people in the

context were so and so. Taken this way, example (25) amounts to the following reformulation.

1 Many people said that Gordon Brown was going to resign.

2 Therefore, Gordon Brown was going to resign.

Formulated so, it is clear that the argument is not valid because (c) no matter how many people say something, it can still be a rumour. We certainly need more evidence to support that what is said is true. For example, whether the utterers were credible, whether they had reliable sources of information, what relationship they had with the subject matter, whether there was any direct, objective evidence to the subject matter, such as whether Gordon Brown himself exhibited any sign of resigning, etc. Testimony is not in general a most reliable evidence of knowledge.

> Example (26): People in a gang, especially young people's gangs, tend to imitate each other's behaviour. They may dress similarly or act similarly. A member may explain: 'Everybody else is doing it, so I thought it'd be a good idea too. It's high fashion.'

Following fashion is perhaps a natural tendency for social animals like human beings. The question is whether it really is a good idea. For instance, people may follow others in smoking, taking drugs, hurting themselves or even harming others, which do not seem to be good. So, whether an action is good or not is to be judged on some independent criteria, not merely because it is done by others.

One may argue that similar behaviours forge identity and cohesion, so it is not necessarily bad. Again, that really depends. Sometimes too much cohesion is not a good thing but creates prejudice or groupthink. **Groupthink** is a type of phenomena studied in social psychology and organizational behaviour in which a group values harmony and coherence over accurate analysis and critical evaluation, which results in an irrational or dysfunctional decision-making outcome. A famous example would be the attack on Pearl Harbor in which the Americans did not believe that the Japanese dared, even though Japanese

messages were intercepted. Groupthink fallacy is usually accompanied by strong emotions; hence it is also classified under the appeal to emotion (see the next fallacy entry).

Once again, appeal to common practice is a common strategy in advertising.[4] Advertisements may refer to surveys, interviews, statistical figures, etc. to make their products look attractive. There are of course questions about genuineness and reliability of the data. On top of that, they create psychological effects like the herd response. We should be aware that the products are not necessarily good just because they are popular.

A related type of fallacy is appeal to populace. **Appeal to populace** is similar to appeal to common practice except that the latter emphasizes more the action taken by people, while appeal to populace may purely be opinion and comments. Populism has taken a new form in the digital age and appeal to populace can easily be found in social media. In view of the ease and speed of communication, even strangers can now easily comment on something on the internet, without showing their identity. Comments tend to be short; sometimes even only symbols and icons are employed. Often people just show their stance without explaining or debating any reasons behind it. Information is filtered, so people in a certain network will only hear like opinions which are not balanced by other ideas and thoughts. This may have a major effect on politics, moral discussion, social division and segregation, and even lead to cyber bullying.

Appeal to common practice is not a sufficient reason to support the truth of the conclusion. Sometimes, it can even give rise to irrational and false decision-making. We thus have to be careful about this kind of argument.

▸ Appeal to emotion (argument *ad populum*)

Appeal to emotion occurs when careful reasoning is replaced with devices to create enthusiasm and emotional support for the conclusion advanced. Specific types include: **appeal to pity**, **apple polishing**, and **groupthink fallacy**.

Example (27):

Student to his tutor: 'I deserve an A for this assignment because I've broken my wrist while typing it.'

(27) is a typical example of **appeal to pity**. The student asks for a good grade not based on his effort and achievement but for something else, in particular his unfortunate circumstances. The aim is to arouse emotion rather than to argue rationally.

Apple polishing is the strategy where we praise someone in order to convince them to do the thing that we want.

Example (28):

Salesperson: Excuse me... I knew as soon as you walked into the showroom that *you will accept nothing but the best*. Let me show you the product...

By praising them first, the salesperson may make a good impression on the customer, who may then be more willing to accept the salesperson's further recommendations. Apple polishing is not wrong as a marketing strategy; it simply is a method of persuasion rather than rational argumentation, and so it would be fallacious to use it in argument.

We already mentioned **groupthink** in the appeal to common practice. The groupthink fallacy is also often a fallacy appealing to emotion because a reasoner may substitute pride of membership in the group for reasons to support the group's policy. 'Blind' patriotism is a rather nasty version of this fallacy. 'Blind' loyalty can also be found to religions, organizations and companies. The following is an example.

Example (29):

Football fans: We know that other teams are never better than ours no matter what. Well, they are not us, are they?

The football fans dismiss other teams as less good just because they believe their own team is the best no matter what. This is not appealing to an objective investigation of the evidence. Thus, it is not a rational argumentation but fallacious reasoning.

▶ Appeal to force (argument *ad baculum*)

Appeal to force occurs when careful reasoning is replaced with direct or insinuated threats to force the acceptance of the conclusion. This is obviously fallacious reasoning, for it is not even a rational discussion. Threats and violence however can come in subtle forms.

> Example (30):
>
> David: My father owns the company that gives your newspaper 20 per cent of all its advertising revenue, so I'm sure you won't want to publish any story about my vandalism.
>
> Newspaper editor: Yes, I see your point. The story really isn't newsworthy.

Whether a story is newsworthy should depend on the editor's journalistic professional judgement, not on business sales. Yet in this case, the subtext of David's utterance is that he would ask his father to stop advertising in the newspaper if the editor reported his criminal acts. This is a threat even though the threat is only implied in the conversation, so David still commits the fallacy of appealing to force in his reasoning that his crime is not newsworthy.

▶ Argument against the person (argument *ad hominem*)

When you attack your opponent's *character* rather than his or her views, you commit the fallacy of argument against the person. The Latin term *ad hominem* means 'to the man'. Whether an argument is good or bad should depend on the truth of its content rather than *who* makes the argument. However, caution should be exercised to distinguish this fallacy from challenges to the integrity of a speaker, which is sometimes relevant when determining whether their testimonials are trustworthy. We shall discuss this in detail below.

Arguments against the person are further divided into several categories.

Abusive *ad hominem*

When the argument is *directly* against persons, seeking to defame or discredit them. It is also called **genetic fallacy**, because it attacks at the genesis of an idea rather than the truth of its content.

Circumstantial *ad hominem*

When the attack is *indirectly* against persons, suggesting that they adopt their views chiefly because of their special circumstances or interests.

Tu quoque (you too)

This is a special type of *ad hominem* argument, in which a person attempts to discredit an opponent's view by pointing out that the opponent does not always act on it. But the fact that a person is *inconsistent* does not mean that what he or she says is false.

The general format of an abusive argument against the person (i.e. abusive *ad hominem*) is as follows: a person has such-and-such negative feature; therefore, his claim (belief, opinion, theory, proposal, etc.) stands refuted. For example:

> Example (31): What Mary says about Johannes Kepler's astronomy of the 1600s must be just so much garbage. Do you realize she's only 14 years old?

Mary might be young, yet she could be right about Kepler's astronomy. For example, she could be well-educated and well-informed in astronomy, perhaps even a genius in the area. Whether a person's opinion is correct depends on the truth of the content but is irrelevant to and independent of the person's age, upbringing, or other background.

One may argue however that we do judge people from certain backgrounds as untrustworthy by our experience. There are several dangers in such ways of thinking. First, our impressions may be biased. People tend to make over-generalizations or under-generalizations depending on their personal experience, pre-established judgements and beliefs, media coverage, which in turn vary according to time and availability, etc. Although we all act

on our perception, it does not imply that our perception is true. Stigma, prejudice, stereotypes can so easily be formed under the same process of perception. Things are not necessarily what they seem to be. Therefore, we should not allow our judgements to prevail over objective facts and evidence of individual cases. Second, even if a person is in general untrustworthy, he or she may still say the truth in particular cases.

Having said that, however, there are indeed occasions where consideration of the person is relevant and legitimate. For example, integrity is important for witnesses testifying in court. Lawyers may show evidence that a witness is incoherent or used to lie to weaken the strength of his or her testimony. The arguments may take the following forms:

1 Colin lies, therefore his testimony is not to be trusted.

2 Colin used to lie, therefore his testimony is not to be trusted.

It is better that the lawyer finds evidence to show the witness lies in the particular case under trial. Failing that, showing evidence that he used to lie will also help weaken the case; i.e. using (2) to suggest (1). In either case, these are indirect arguments against what is said in the witness's testimony. What is said could be the truth even though the witness may not see it himself and hence his testimony is false. That is why in general, testimony is not a very reliable source of evidence. However, since there may not be any other stronger evidence available and the court has to rely on testimonies, the court has to consider indirect arguments for whether the testimonies are trustworthy, to pick up the best among the worst or cast out any reasonable doubt. This is to be prudent. The reasoning is not that because the witness is unreliable, therefore his testimony is false. Rather, it is simply that because the witness is unreliable, it is reasonable for the court not to trust his testimony. True or false is one thing; trustworthy or dubitable or not, is quite another.

Another occasion in which the question of the integrity of the speaker legitimately matters is during political campaigns. For instance, immoral sexual conduct is irrelevant to the quality of a person's mathematical reasoning. However, it is

relevant to arguments promoting the person for a leadership position in the Church or in the state. This is because, arguably, political leadership does require moral leadership. A state exists to help its citizens to live a good life. Hence, the leader of a state does not only need to have the necessary skills to run various administrative and financial functions of an organization, he or she also needs the vision and the moral commitment to what a good life means. Yet one cannot truly know what a good life is without being committed to practising it. Hence, a good political leader should also have certain moral qualities.

The second type of argument against the person is the circumstantial argument against the person. As the term suggests, the argument is not directly about the person who makes the argument. Rather, it points to the background, circumstances and interests of the person, not necessarily for its truth but for other reasons, such as ulterior motives.

> Example (32): Don't trust Friedman's economic theory. He is a millionaire, so of course he would support tax cuts.

Example (32) is an argument against the person because it is not based on criticisms of or comments about the economic theory itself but on the person who proposed it, namely, Friedman. It is circumstantial, not abusive, because it does not directly attack Friedman's character or other personal features. Rather, it focuses on his background. It is fallacious because even if it is in Friedman's personal interest to enjoy tax cuts, it does not imply that he supports it because of such interest. Whether Friedman's economic theory stands is to be judged by academic scrutiny, empirical research, data support, etc. It has nothing to do with the author's personal background.

The last type of attacking the person argument we will look at is the *tu quoque* (you too) argument. It is an argument in which the reasoner attempts to discredit his or her opponent's views by pointing out that the opponent does not always act on them. It is fallacious because even if a person is inconsistent, it does not follow that what he or she says is false.

Example (33):

Father: You should not lie.

Son: You can't accuse me of lying; you lie too!

We can reformulate the son's argument as follows.

1 The father lies.

2 The father does not act on his claim that lying is wrong.

3 Anyone who claims X but does not act on X cannot accuse others for not following X.

4 Therefore, the father cannot accuse others (the son) for lying.

The argument so formulated is formally valid. Yet the question is why (3) should obtain. Anyone who claims X but does not act on it is inconsistent in character at most. It does not imply that what he claims is false. It seems that (3) must be cashed out further.

1 The father lies.

2 The father does not act on his claim that lying is wrong.

3' Anyone who claims X but does not act on X is inconsistent.

4' If a person is inconsistent about X, the person does not believe in X.

5 The father is inconsistent about lying being wrong.

6 Therefore, the father does not believe that lying is wrong.

7 One cannot accuse others of something one does not believe in.

8 Therefore, the father cannot accuse others (the son) of lying.

Now the problem is the truth of (4'). A person can indeed be right about something yet inconsistent in carrying it out. This happens all the time in ethics. It is called weakness of the will: one believes something is good and one wants to do it, yet somehow one is unable to resist the temptation to do otherwise or is simply not committed to doing it. It is thus wrong to infer that a belief is false just because its holder cannot follow it through.

Tu quoque arguments can appear in more serious and public matters. Consider the following.

Example (34): We can disregard all Jefferson's fine words about human equality. Don't you know that he was a slaveholder himself?

Thomas Jefferson (1743–1826) was the third American president and a principal author of *The Declaration of Independence.* The above argument can be reconstructed as follows.

1 Jefferson claimed that all humans are equal.

2 Jefferson did not act on the view that all humans are equal.

3 If someone claims X but does not act on X, then X is false.

4 Hence, Jefferson's view that all humans are equal is false.

Taken like that, it is obvious that premise (3) is suspect. When someone does not act on what he claims, it means that person is weak in his will, has some character flaw, or has some acute circumstantial reasons preventing him from being what he would like to be. It does not imply however that the belief so claimed is false. We may want to find out the reasons for the inconsistency, or legitimately cast doubts on the integrity of the person. However, whatever the person may be, it still does not imply what is said is false.

Another case for thought is the war on terrorism in recent decades.

Example (35): The USA has no right to condemn terrorism. It itself kills innocent civilians in air-raids and bombings.

Problems related to this example include:

1 What terrorism is – is killing innocent civilians in air-raids and bombings sufficient to be called terrorism?

2 What is meant by someone having a 'right to condemn terrorism' – in the most extreme case, can a terrorist himself condemn terrorism? Can someone deliberately do something one considers immoral?

3 What is the real aim of this argument? What is the role of the USA in global efforts against terrorism? Does this argument challenge the moral quality of American leadership?

I leave this discussion for readers to ponder further. This has been a long section, so let's take a break and review what we have learned.

Key idea: Fallacies of appeal

Appeal to ignorance: When it is argued that a proposition is true simply on the basis that it has not been proved false, or that it is false because it has not been proved true.

Appeal to inappropriate authority: When an argument appeals to a party having no legitimate claim to authority in the matter at hand.

Appeal to common practice/populace: When a speaker appeals to the fact that a view is universally or commonly held in support of it.

Appeal to emotion: When careful reasoning is replaced with devices to create enthusiasm and emotional support for the conclusion advanced. Specific types: appeal to pity, apple polishing, groupthink fallacy.

Appeal to force: When careful reasoning is replaced with direct or insinuated threats to force the acceptance of the conclusion.

Argument against the person (argument *ad hominem*):

Abusive *ad hominem*: When the argument is *directly* against persons, seeking to defame or discredit them. It is also called the genetic fallacy.

Circumstantial *ad hominem*: When the attack is *indirectly* against persons, suggesting that they adopt their views chiefly because of their special circumstances or interests.

***Tu quoque* (you too):** This is a special type of *ad hominem* argument, in which the reasoner attempts to discredit the opponent's views by pointing out that he or she does not always act on them.

FALLACIES ABOUT PREMISES AND CONCLUSION

Fallacies of relevance include fallacies of appeal, fallacies about premises and conclusion, and fallacious induction. Now let us move on to the other two types.

Fallacies about premises and conclusion focus on how the premises and the conclusion are not appropriately related in structure.

▶ Complex question

The argument assumes some unstated premises. Yet with such missing unstated premises, it misleads the audience into believing something which the responder cannot clarify by giving a simple answer. So, although the question may look naïve, it actually involves a complex twist. It is fallacious reasoning in the sense that it leads the audience into believing something while it does not give a fair chance for the responder to notice the implicit assumption and clarify.

Example (36): Do you still beat your wife?

The question already assumes that the responder beat his wife in the past. If the responder answers 'yes', he admits that he beat his wife in the past and still does so now. If he answers 'no', he admits that he beat his wife in the past but has stopped doing it now. Either way, it leads the responder into the trap of admitting that he beat his wife in the past. It is not good reasoning because it does not reveal its assumption explicitly and give the responder a fair chance to clarify. Perhaps the responder has never beaten his wife; the question however conceals such a possibility.

Sometimes, however, it is exactly this result that the interrogator tries to exploit: luring the responder into admitting something he or she probably would deny if asked outright. This could be done for a good cause, say, to reveal some scandals or secrets. Yet it could also be done with bad motives.

Example (37):

Reporter: Mr Prime Minister, can you tell us why you keep wasting taxpayers' money on refugee issues?

What is taken for granted in this reporter's question? What would the prime minister have admitted in giving any answer to this question? Yes, the unstated premises are that (1) it is a waste of taxpayers' money to spend on refugee issues; (2) the prime minister has wasted money in this way before. No matter the answer, the prime minister would have admitted such assumptions. Whether it is a good or bad thing to get the prime minister to answer that way depends on the context and the purpose of the reporter.

In any case, the general aim in reasoning is to make the argument clear and transparent. For exactly such purpose, complex question is not a good strategy.

▶ Begging the question (*petitio principii*)

An argument **begs the question** when the reasoner assumes in the premises the truth of what he or she seeks to establish in the conclusion. In other words, the premises can only be true if the conclusion is true. Yet normally, an argument is supposed to infer from the truth of the premises the truth of the conclusion, so the truth of the premises is independent of, and logically prior to, the truth of the conclusion. This fallacy violates such an idea and so is fallacious.

> Example (38):
>
> A: Computers can't think.
>
> B: Why?
>
> A: It is because if computers can think, they must first have a mind. But you know that computers don't have minds!

We can reformulate A's argument in the following way.

1 Only things that have minds can think.

2 Computers do not have minds.

3 Therefore, computers cannot think.

Suppose (1) is true and not to be disputed. However, it is far from obvious that (2) is true without already assuming that (3) is true. It is because a common criterion for determining whether a certain thing has a mind is to see whether it can think. René Descartes (1596–1650), for instance, claimed animals do not have minds because they cannot think or reason. I do not endorse this view. I simply want to point out that if having a mind and being able to think are not conceptually separate from each other, then in asserting (2), the reasoner would have already assumed (3). Yet an argument is supposed to support a conclusion based on reasons independent of the conclusion. This argument implicitly uses (3) to support (2). So, it begs the question.

We can determine whether a machine is intelligent prior to determining whether it can think or has a mind. Since thinking is a sign of intelligence, some may argue that a sufficiently intelligent machine indeed thinks. Alan Turing (1950) designed a test of a machine's ability to exhibit intelligent behaviour equivalent to, or indistinguishable from, that of a human, called the Turing Test. Functionalism, a theory in the philosophy of mind, argues that a mind should be identified by its function, rather than what it is made of. Thus, if two objects exhibit the same function, then they should be granted the same status. It follows that if a machine performs with exactly the same intelligence as a human, then it is possible that it has a mind.

▶ Circular argument

A **circular argument** is an argument in which the premises do not only *support*, but are in turn *supported by* the conclusion. Circular arguments are very close to begging the question. Indeed, any argument that begs the question can be formulated in the form of a circular argument. However, an argument that begs the question may or may not be *actually* worded in a way that looks circular. So begging the question is more about the conceptual relations between the truth of the premises and the conclusion, whereas circular arguments are more formal and syntactic, i.e. the arguments can be formulated in a way which sees the conclusion appear in the premises.

Circular arguments are obviously fallacious because an argument is supposed to move from the truth of the premises to the truth of conclusion, yet circular arguments make it the other way around: the conclusion already appears as one of the premises, so its truth is assumed.

Let us consider (38) again. To make the point clearer, we will simplify the formulation of its argument as follows.

1 Computers do not have minds.

2 Therefore, computers cannot think.

Suppose we ask further: why do computers not have minds? And if we answer: because they cannot think, then the answer implies the following argument.

3 Computers cannot think.

4 Therefore, computers do not have minds.

We see clearly now that (3) and (4) is exactly the reverse argument of (1) and (2). (3) is (2) and (4) is (1). In other words, according to the arguments, the truth of (1) depends on (2) and the truth of (2) depends on (1). (1) and (2) thus form a circular argument.

Let us illustrate this with another example.

Example (39):

A: God exists.

B: How do you know that?

A: Because the Bible says so.

B: Why should we trust the Bible?

A: Because God says we should trust it.

B: How do you know that God says we should trust the Bible.

A: Because the Bible says so.

A's argument is a circular argument. To arrive at the conclusion that God exists, a proper premise should not have already assumed God's existence. Initially, A's premise does not seem to have already assumed his conclusion, because A does appeal to something else, which is the authority of the Bible, to establish God's existence. However, the authority of the Bible is in turn established on the assumption that God exists. So, indeed it is a circular argument.

The last two lines of the dialogue make the circularity even more apparent. So, forget about God's existence, after all, all evidence for the Bible's claim to authority is that it says it has the authority. No external evidence is given. If one does not already believe that the Bible contains God's words, what it says cannot induce one to believe that it contains God's words. The absurdity is thus evident.

▶ Misplacing the burden of proof

The fallacy of **misplacing the burden of proof** occurs when the burden of proving a point is placed on the wrong side. Generally speaking, the burden of proof is on the person whose views go against common sense. Moreover, the person who is asserting the existence of something has the burden of proof, as opposed to those denying the existence.

A classic example is George W. Bush's allegation of Saddam Hussein, which eventually led to the Second Gulf War in 2003.[5]

> Example (40): Saddam Hussein has not demonstrated that he doesn't have weapons of mass destruction. Therefore, he has them.

Since it is not common for countries to have weapons of mass destruction (WMDs), the burden of proof should be on the side which asserts their existence rather than the side to deny it. Bush asserted that Hussein had WMDs. Thus, the USA should prove that WMDs exist in Iraq, rather than the other way around. The statement in (40) misplaces the burden of proof on Iraq. If Iraq really had no WMDs, then it would be strange to ask her to prove something non-existent in the first place.

> Example (41):
>
> Andrew: 'Did you know that, if you rub red wine on your head, your grey hair will turn dark again?'
>
> Bob: 'No way. I don't believe it.'
>
> Andrew: 'Hey, why not? How do you know it won't work?'

In example (41), Andrew argues that rubbing red wine on one's head will turn grey hair dark. This is novel, so Andrew should have the duty to explain and support what he is saying. Yet instead of doing so, he responds to Bob's question by asking Bob to explain his disbelief. This places the burden of the argument on Bob to prove his point, rather than on Andrew to substantiate his claim. Andrew has thus misplaced the burden of proof to others.

It is interesting to compare misplacing the burden of proof with the appeal to ignorance. The appeal to ignorance argues that something exists (or does not exist) because there is no proof otherwise. Misplacing the burden of proof does not produce any proof; indeed, it does not even aim at doing so. Its point is to shift the responsibility and claim that the opponent has to prove their point rather than the proponent. So, the placement of the burden of proof is more like an etiquette problem rather than a problem of substantial content, like the appeal to ignorance.

Once it is decided which side has the burden of proof, if the side required to produce evidence is unable to, they then look vulnerable. For example, extra-sensory perception (ESP) is novel. The burden of proof of its existence should be on those who propose it. If they cannot produce evidence to prove it, then the position is weakened: it is unlikely that ESP exists. However, if they claim ESP exists merely because they cannot disprove its existence, such argument would commit the fallacy of appealing to ignorance.

Because the placement of the burden of proof is an etiquette problem, it is sometimes regarded as a matter of rhetoric or strategy rather than a very serious problem. For example, suppose someone proposes a novel theory and a critic argues that it is not good enough. Common sense has it that the proponent of the theory should explain themselves. Yet if the proponent accuses the critic of being unclear in specifying what kind of evidence is needed and why the existing evidence is inconclusive, then the proponent may have a point in putting the burden back onto the critic to justify their requests. This would then be an endless game of shifting responsibility to prove something rather than bringing the discussion forward on the substantial problem.

▶ Irrelevant conclusion (*ignoratio elenchi*)

Irrelevant conclusion occurs when the premises miss the point, purporting to support one conclusion while in fact supporting or establishing another.

> Example (42): We should cut down the emergency services at hospitals, so it won't be abused.

When emergency services at hospitals are abused, the proper response should be to devise ways of preventing such abuse, or to remedy the situation so those who are in need of the emergency services are not affected. However, in (37), instead of drawing the relevant conclusion, the reasoner proposes cutting emergency services to avoid the abuse. This is like saying, instead of fighting a disease, we will kill the patient so that the disease will die too. This is a bad argument because the conclusion (the suggested solution) is not relevant to the premises (the problem).

FALLACIOUS INDUCTION

The last category of fallacies concerns inductions. A valid deductive argument has its conclusion wholly supported by the premises, namely that if the premises are true and the argument is valid, then the conclusion is guaranteed to be true. In inductions, however, the conclusion of an induction is only partially supported by the premises in that if the premises are true, the conclusion is likely to be true.

Example of deduction:

> Socrates is human.
>
> <u>All humans are mortal.</u>
>
> Therefore, Socrates is mortal.

Example of induction – inductive generalization:

> <u>All surveyed customers are satisfied with the product.</u>
>
> Therefore, all customers are satisfied with the product.

Example of induction – inductive syllogism:

> Most university students oppose budget cuts in higher education.
>
> <u>Sebastian is a university student.</u>
>
> Therefore, Sebastian opposes budget cuts in higher education.

All inductions are invalid; however, they can still be distinguished as strong or weak arguments, namely whether the conclusion is highly likely to be true or rather unlikely if the premises are true. In the examples above:

- The greater the number and diversity of customers surveyed, the higher the probability that the conclusion is true.
- Similarly, the larger the number of university students opposing budget cuts, and the more typical Sebastian is of a university student, then the more likely it is that he opposes budget cuts too.

Fallacious inductions concern weak inductions such that the conclusion is likely not to follow. They are different from composition and division in that the latter are caused by semantic ambiguity of a term applied to a part and the whole, whereas fallacious inductions focus simply on the strength of the connection between premises and conclusion.

Hasty generalization

The fallacy of **hasty generalization** occurs when one moves too carelessly or quickly from a single case to an indefensibly broad generalization.

> Example (43): Mentally ill patients are violent. We have heard many stories about violent patients in TV dramas and on the news, right?

Generalizations are arguments from specific incidents to general patterns. A generalization is fine if it is supported by a significant number of incidents and the incidents are representative of all incidents (e.g. cover a wide variety within the population).

When considering (43), there are actually very few incidents of violent mental patients reported in the media, in comparison to the very large number of non-violent mental patients. However, the latter tend to be ignored or forgotten, whereas the former retain and call for people's attention. Hence, this generalization is too hasty and unreliable.

That we tend to only retain and recall information that is readily available to us recently or that has been boosted via media coverage is called an **availability bias**.

> Example (44):
>
> A fish and chip shop owner: Take my son, Martyn. He's been eating fish and chips his whole life, and he just

had a cholesterol test and his level is below the national average. What better proof could there be than a frier's son?

This example is again a case of hasty generalization. The shop owner uses one case to generalize that eating fish and chips does not cause increased levels of cholesterol. But his son is only one case, and he is no different from any other people who regularly patronize the shop. The argument may have a psychological or rhetorical effect, but it does not make a difference in logic.

▶ Accident

Accident is the fallacy in which one applies a generalization to an individual case that it does not properly govern. So, accident is like the converse of hasty generalization: hasty generalization argues wrongly from individual incidents to a general pattern, whereas accident argues wrongly from a general pattern to individual cases.

> Example (45): People should keep their promises, right? I've loaned David my knife. So he should keep his promise and return it to me so I can stab my neighbour with it.
>
> Example (46): Many rich people live in the USA. Bill Gates lives in the USA. So, Bill Gates is rich.
>
> Example (47): We should love our country. Therefore, we should obey its leader all the time.

It seems clear that, while the general statements in the above examples are fine, the conclusions drawn are wrong as they involve inappropriate applications of the general statements. In all these cases, if there is any connection between the general statement and the individual incident, it would be a contingent accident that they are so linked, rather than having any necessary or intrinsic linkage.

In (45), keeping a promise is morally right, yet facilitating harm is not. Similarly, although many rich people live in the USA, it does not imply someone is rich just because they live in the USA. Bill Gates is indeed rich, but not because he lives in a rich country. So (46) is still fallacious.

In (47), patriotism may or may not be a virtue. Yet even if it is, it does not imply that a person should obey the leader of their country blindly. A country is a collective composed

of all citizens with a history and culture of its own. Political leadership changes all the time yet the country carries on. Individual leaders may make wrong decisions. It is indeed a citizen's duty to monitor and criticize what the government is doing to ensure that its policies are in the best interests of the country.

▶ False cause

False cause is a fallacy in which one treats as the cause of a thing that which is not really the cause of the thing.

> Example (48): Every time Henry saw a black cat before entering a casino, he lost. So seeing a black cat is the cause of his loss.

(48) argues that the appearance of a black cat is the cause of Henry's loss. Yet there can't be any inherent correlation. The appearance of a cat is quite random and accidental. Suppose Henry did lose every time he saw a black cat before entering a casino; the best we can do is to establish a statistical correlation. However, this may still be hasty generalization depending on how many times Henry entered a casino and how often he saw a black cat and lost at the same event. Not to mention that a cause is much more than a statistical correlation. There should be some necessary links between the two events (cause and effect) rather than the mere fact that they regularly come together. To use an example by David Hume, day and night always come together, but day does not cause night, nor does night cause day.[6]

We can compare this example with the **gambler's fallacy**, in which the reasoner falsely assumes that the past outcomes will affect future outcomes when in fact the events are independent of each other.

> Example (49): This coin has come up heads five times in a row now, so the next toss must be tails.

Assuming a fair coin is used, the chance for heads or tails should be equal and each flipping is an independent event from the next. This means that tossing five heads in the past does not have any bearing or influence on the result of the next toss. It is just wishful thinking that people expect the next outcome to be affected by the past.

We have learned some more fallacies in the last two sections. Let us sum up and do some exercise.

Key idea: Fallacies about premises and conclusion

Complex question: The argument assumes some unstated premises.

Begging the question: When one assumes in the premises the truth of what one seeks to establish in the conclusion.

Circular argument: The premises do not only *support*, but are in turn *supported by* the conclusion.

Misplacing the burden of proof: When the burden of proving a point is placed on the wrong side.

Irrelevant conclusion: When the premises miss the point, purporting to support one conclusion while in fact supporting or establishing another.

Key idea: Fallacious induction

Accident: When one applies a generalization to an individual case that it does not properly govern.

Converse accident/hasty generalization: When one moves carelessly or too quickly from a single case to an indefensibly broad generalization.

False cause: When one treats as the cause of a thing what is not really the cause of that thing.

Exercise 3.2: Informal fallacies (2)

Identify the fallacies, if any, in the following sentences and explain them:

1 But can you doubt that air has weight when you have the clear testimony of Aristotle affirming that all the elements have weight including air and excepting only fire?

2 To ignore the possibility that America was discovered by Africans because these explorers are 'unknown' is irresponsible and arrogant. If we are unaware of an event, does that mean it never happened?

3 Logicians claim that the study of argument is an indispensable part of everyone's education. Since this helps to keep them employed, it's not surprising that they endorse such views. Let's not be influenced by thinking which does not represent the majority of students, parents, or teachers themselves.

4 How dare you criticize my logic? You commit fallacies every time you write a paper or sit an exam.

5 Fallaci wrote her: 'You are a bad journalist because you are a bad woman.' (Elizabeth Peer, 'The Fallaci Papers', *Newsweek*, 1 December 1980)

6 A: Ghosts exist.
B: Nonsense!
A: What do you mean 'Nonsense'? Can you prove that ghosts *don't* exist?

7 An attorney is always free to consult law books. And a physician often looks up cases in medical texts. Everyone should be allowed a similar freedom of reference. So students should be permitted to use their textbooks during examinations.

8 Everything has a cause because everything is the effect of something else.

3.3 Chapter summary

To sum up, there are many fallacies, formal or informal, discussed by philosophers and logicians. Fallacies are discussed to highlight common pitfalls in our reasoning. In this chapter we discussed some informal fallacies before going through the formal part of the book, in order to foster a heightened sense

of critical thinking – arguments are not just about being valid or invalid; sometimes problems arise before we even need to examine their abstract forms.

In this chapter we introduced altogether 25 informal fallacies. We categorized them into two groups: fallacies due to the abuse of language and fallacies of relevance. The latter was further categorized into three types: fallacies of appeals, fallacies about premises and conclusions and fallacious induction. The following table highlights their main features.

Name and type	Definition	Example
Fallacies due to the abuse of language		
Equivocation	When the same word or phrase is used with two or more meanings, deliberately or accidentally, in the formulation of an argument.	Time is money. Time can heal all wounds. Hence money can heal all wounds.
Amphiboly	When a statement in an argument has more than one plausible meaning due to ambiguous grammar.	Oracle: 'If the King of Lydia went to war with Cyrus, he would destroy a mighty kingdom.'
Accent	When a shift of meaning arises within an argument as a consequence of changes in the emphasis given to its words or parts.	'We should not speak *ill* of our friends.' 'We should not speak ill of *our* friends.'
Slippery slope argument	A small concession is presented as having potentially catastrophic consequences.	Bureaucrat: 'Oh, we cannot entertain this request. For if everyone makes the same request, we shall not be able to handle them all and the whole system will collapse.'
	Compare slippery slope argument to the sorites paradox: they both make use of vagueness of words.	Abortion is wrong because it is surely wrong to kill a newborn baby, and if it is wrong to kill a newborn baby then it is wrong to kill a foetus on the eve of its birth because the two are biologically very similar. Similarly, it is wrong to kill a foetus two days prior to its birth because a foetus one day prior to its birth is so similar to a foetus two days prior to its birth. Because there is no point at which the foetus is essentially different from the moment before, there is no time at which aborting a foetus is not wrong.

(continued)

Name and type	Definition	Example
False dichotomy/ false dilemma	Two alternatives are presented as exhaustive, when in fact other possibilities exist.	If you eat too much, you'll be stuffed to death. If you eat too little, you'll starve to death. So no matter how much you eat, you'll die.
Straw man fallacy	When you level your arguments against a crude caricature of your opponents' views (you set up a 'straw man' to knock down but in fact leave the real issue unresolved).	'We should not discriminate against homosexuals.' 'So are you saying that we should all be gay?'
Red herring	Changing the subject, diverting attention away to other issues.	'What's your view on recent political developments?' 'The weather is fine today.'
Composition	When one mistakenly reasons from the attributes of a part or an individual to the attributes of the whole or the totality of that collection.	Every footballer is highly skilled individually; therefore, the team they formed must also be good.
Division	When one mistakenly reasons from the attributes of a whole or a totality of the collection to the attributes of its parts or individual members.	The USA is an affluent country; hence, every person living there is rich.
Fallacies of appeal		
Appeal to ignorance	When it is argued that a proposition is true simply on the basis that it has not been proved false, or that it is false because it has not been proved true.	God exists because you cannot prove that it doesn't.
Appeal to inappropriate authority	When an argument appeals to a party having no legitimate claim to authority in the matter at hand.	We are not morally wrong in bullying the weak because Darwin said that evolution relies on survival of the fittest.
Appeal to common practice	When a speaker appeals to the fact that a view is universally or commonly held in support of it. Yet a view can be commonly held and nonetheless false.	It is morally fine to eat meat because everyone is doing so. Vegetarianism is nonsense.

(continued)

Name and type	Definition	Example
Appeal to emotion	When careful reasoning is replaced with devices to create enthusiasm and emotional support for the conclusion advanced. Specific types: appeal to pity, apple polishing, groupthink fallacy.	Our team will win because it is *our* team!
Appeal to force	When careful reasoning is replaced with direct or insinuated threats to force the acceptance of the conclusion.	Be careful what you say; you know who is listening and if you say something opposing the authorities, you know the consequences...
Abusive *ad Hominem*	When the argument is directly against persons, seeking to defame or discredit them. It is also called genetic fallacy.	What Tom says must be wrong; you know he is an idiot.
Circumstantial *ad Hominem*	When the attack is indirectly against persons, suggesting that they adopt their views chiefly because of their special circumstances or interests.	Friedman's economic theory is biased in favour of the rich because he is a millionaire himself.
Tu quoque (you too)	When you attempt to discredit your opponent's views by pointing out that he or she does not always act on them.	'You can't accuse me of lying; you lie too!'
Fallacies about premises and conclusion		
Complex question	When the argument assumes some unstated premises.	Do you still beat your wife?
Begging the question	When one assumes in the premises the truth of what one seeks to establish in the conclusion.	A computer can't think because it does not have a mind.
Circular argument	When the premises do not only support, but are in turn supported by the conclusion.	God exists because the Bible tells us so. And why should we trust the Bible? Because God tells us so.

(*continued*)

Name and type	Definition	Example
Misplacing the burden of proof	When the burden of proving a point is placed on the wrong side. Generally speaking, the burden of proof is on the person whose views go against common sense.	Saddam Hussein has not demonstrated that he has no weapons of mass destruction. Therefore, he has them.
Irrelevant conclusion	When the premises miss the point, purporting to support one conclusion while in fact supporting or establishing another.	Father: 'You shouldn't go out wearing so little.' Daughter: 'So are you saying I should not go out at all?!'
Fallacious induction		
Accident	When one applies a generalization to an individual case that it does not properly govern.	We should not lie. Therefore, we should not lie to save lives.
Converse accident/ hasty generalization	When one moves carelessly or too quickly from a single case to an indefensibly broad generalization.	Toyota are good cars, because a friend of mine has one and it is good.
False cause	When one treats as the cause of a thing what is not really the cause of that thing.	Depression often comes with sleeping disorder. Hence, sleeping disorder is the cause of depression.

So much for informal fallacies. This concludes our discussion of informal matters related to critical thinking and reasoning. In the next chapters we will introduce formal logic properly and discuss the three logic systems: categorical logic, propositional logic and predicate logic. The first one is categorical logic.

4

Categorical logic

In this chapter you will learn about:

- ***propositional forms – AEIO***
- ***translating ordinary language into categorical propositions***
- ***immediate inferences and the traditional square of opposition***
- ***existential import***
- ***using Venn diagrams to test the validity of arguments***
- ***the rule method***

> *'How often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth?' (Sherlock Holmes)*
>
> Arthur Conan Doyle (1890), *The Sign of the Four*, Chapter 6

4.1 Propositional forms – AEIO

Categorical logic is one of the oldest surviving systems of logic which is still widely used in contemporary life. It dates back over 2000 years to Aristotle in Ancient Greece. As can be expected of anything that has lasted so long, different systems of presentation and interpretations have been introduced over the years. This chapter introduces two types of understanding: the traditional one and the one presented by the Venn diagram. The latter has transformed the former on the issue of existential import and has become the standard for modern understanding of categorical logic.

To start understanding how categorical logic works, let us first introduce some features of logic in general.

Any system of logic is composed of three elements:

- **Semantics**: specifies the set of symbols to be used in the system and what they stand for
- **Syntax**: specifies what types of combinations of symbols are regarded as well-formed
- **Rules of inference**: govern valid inferences between well-formed formulae.

To grasp a system of logic, we need to understand all three aspects of the system. In the following chapters we will introduce the semantics and syntax of a logical system, followed by how that logic determines validity of arguments. While some logic determines validity by literally applying rules to examine the reasoning (such as the rule method in categorical logic and natural deduction in propositional logic), many modern methods of testing validity do not explicitly appeal to logical truths as rules but only establish procedures

to guide through the tests. In learning the constitutive rules of these methods (such as the Venn diagram or truth-table methods), we acquire the ability to test all possible arguments. Therefore, no knowledge of any substantial rules is needed.

Different logic systems have different semantics, syntax and rules of inference. We can compare these aspects with those of a natural language. Semantics is comparable to the vocabulary of a natural language. Syntax is comparable to the grammar. The rules of inference are the tools or methods by which validity of arguments is proved in the system. It is preferable that the semantics of a logic system correspond with the features of natural language as far as possible, so it is easy to apply the said logical system to analyse our natural reasoning. However, this is not always achievable because, after all, a logic language is an artificial language and natural languages are usually much messier and more complicated than purpose-built artificial languages. Moreover, logicians are more concerned with the system's formal properties such as coherence, consistency and completeness than its applicability.

Categorical logic analyses the *internal* structure of a sentence in a language. Its unique insight is that a simple proposition is composed of a subject–predicate structure. Each subject term and predicate term represents a set of objects, and the verb acts as a bridge (the copula) to link the two sets. So, a simple proposition essentially asserts a relation between two sets of objects. To illustrate, a simple English sentence has the basic structure as follows:

> Subject + verb + predicate
>
> (Example: Snow + is + white)

Whereas the logical structure of a simple proposition in categorical logic is as follows. Note the striking similarities between the two.

> Subject + copula + predicate
>
> (a set of objects + link + another set of objects)

According to categorical logic, the copula has two features: quality and quantity. **Quality** refers to affirmation or negation, meaning whether a set of objects overlaps with another set of objects, or not. The quality is indicated by the choice of

copula in forming a proposition: 'is' indicates affirmation; 'is not' indicates negation. Note that agreement in singularity or plurality is not a concern in logic. There is no difference between using 'is' and 'are' or 'is not' and 'are not'. For convenience, we will use the singular.

Quantity refers to how many objects, or to what extent the set of objects in the subject term, are asserted. We add a quantifier at the front of a sentence to reflect this property. 'All' or 'no' indicates universality; 'some' indicates particularity. Hence, a full exposition of the logical structure of a simple proposition is as follows,

> Quantifier + subject + copula (indicating the quality) + predicate

Quantification may be universal or particular, represented by words like 'all', 'no' (for universals) and 'some' (particular). The assertion of linkage can be affirmative or negative, represented by words such as 'is' (affirmative) and 'is not' (negative). There are only four combinations of quality and quantity, thus generating four types of propositions. Categorical logic recognizes *only* these four types of propositions as well-formed. These four proposition forms are traditionally represented by four vowel sounds called the A-proposition, the E-proposition, the I-proposition and the O-proposition.

	Affirmative	Negative
Universal	*A* (All S is P)	*E* (No S is P)
Particular	*I* (Some S is P)	*O* (Some S is not P)

Example of corresponding categorical propositions:

> A-proposition: 'All politicians are men.'
>
> E-proposition: 'No politicians are men.'
>
> I-proposition: 'Some politicians are men.'
>
> O-proposition: 'Some politicians are not men.'

Categorical logic recognizes only these four proposition types and is designed to handle them exclusively. Any other type of

proposition is beyond its scope. Thus, if a sentence in a natural language is more complicated than this, then it first has to be translated into one of the four standard forms before we can apply categorical logic to analyse arguments concerning it.

To summarize, categorical logic has the following semantics and syntax.

▶ Semantics

The following vocabulary is used:

- Sets of objects (represented by capital letters): S, P, M, or A, B, C...
- Copula: 'is', 'is not'
- Quantifiers: 'all', 'no', 'some'

▶ Syntax

Only four types of propositions are recognized as well-formed in categorical logic:

- A-proposition: All S is P
- E-proposition: No S is P
- I-proposition: Some S is P
- O-proposition: Some S is not P

TRANSLATING ORDINARY LANGUAGE INTO CATEGORICAL PROPOSITIONS

The more a logic language resembles natural language, the more powerful its applicability is. When we compare the semantics and syntax of categorical logic with natural languages, obviously often those languages are a lot more varied. This does not preclude us from making use of categorical logic though. We just have to translate natural language sentences into one of the A, E, I, O proposition forms to make categorical logic applicable to them. Some of such translations are quite natural and intuitive. Some, however, may look a bit artificial.

▶ Adjectives

Adjectives qualify objects. Categorical logic deals with relations between sets of objects. It thus seems intuitive to translate an adjective 'X' into a noun in the form of 'X objects'. For example:

> Example (1):
>
> All oranges are sweet.
>
> All oranges are sweet objects.

▶ Verbs

Some sentences do not use the verb 'to be'. However, it is still obvious that the verb suggests an affirmative or a negative relation and a property to which the subject term is predicated. We extract these two logically relevant elements and rewrite them into the standard form of a categorical proposition. For example:

> Example (2):
>
> Some lions run fast.
>
> Some lions are objects that run fast.

▶ Alternative quantifiers

It seems fairly intuitive that the following pairs of quantifiers express the same quantification relation.

- Every = All
- None = No
- Most, a few, several, the majority/minority of … = Some

For example, 'every day is boring' shares the same meaning as 'all days are boring days'.

▶ Missing quantifier

In ordinary language, sentences often do not have explicit quantifiers. In such cases, we need to interpret in context what the sentence really aims to assert and add the suitable quantifier.

Example (3):

Snow is white.

Snow is white object.

All snow is white object.

In (3), we need to not only translate the adjective 'white' into 'white object', but also interpret the quantity of the sentence. In this context, it is reasonable to assume that the sentence is generally about all snow rather than a particular flake of snow. Hence, a universal quantifier is appropriate. Because it is an affirmative sentence judging from the use of 'is' rather than 'is not', the sentence should be formulated as an A-proposition.

It would not matter for the purpose of logic whether certain grammatical rules are followed, such as agreement in quantity or tense. Some natural languages such as Chinese are not sensitive to such formats. What matters is to show the correct quality and quantity of the proposition.

▶ 'Only'

In translation, we need to preserve the exact logical relation the sentence conveys. Consider the following example. Should (4) be translated as (4a) or (4b)?

Example (4): Only men philander.

(4a) All men are philanderers.

(4b) All philanderers are men.

(4) does not claim that all men have multiple romantic relationships. Rather, it expresses that philanderers are restricted to men, namely, we can only find philanderers among men. Hence the set of objects being quantified is actually philanderers but not men. If (4) is true, (4b) rather than (4a) would be the case. Hence, (4b) is the correct translation of (4).

The example represents a general situation. The word 'only' indicates that the set of objects being universally quantified is

the predicate term, rather than the subject term. Hence, we get the following understanding:

Only S is P = All P is S

▸ Parameters

Some sentences do not mention objects explicitly. Yet categorical logic examines relations between sets of objects, and we thus need to assign sets to the sentence perhaps a bit more artificially by adding suitable parameters. For example, (5) can be translated as either (5a) or (5b).

Example (5): Every time I see this movie, I cry.

(5a) All the times when I see this movie are the times when I cry.

(5b) All occasions on which I see this movie are occasions on which I cry.

Although 'I' is mentioned in (5), the subject matter is not really about me but only the events I experience. To ascribe these experiences as sets of things, we should add some parameters; in this case either 'times' or 'occasions' or even 'events' is appropriate. We may similarly construct sentences with words like 'whoever', 'whatever', 'whenever', 'wherever', etc., as being about 'all persons', 'all things', 'all times', 'all places/locations', etc., respectively.

▸ Singular terms

Singular terms are terms referring to a single particular object, such as proper names. They represent an interesting case in logic because they ascribe both universal and particular relations. Let me explain.

We need to take two steps in translating propositions with singular terms. Consider (6). We can easily translate the predicate term 'is wise' as 'is a wise person'. However, how do we understand the subject term 'Socrates'? There is one and only one Socrates in the whole world. If Socrates is an element of a set of objects, then that set of objects would just contain one

object, namely Socrates himself. (6) can then be understood as expressing (6a). We call a set that contains exactly one element a singleton. Yet what quantity is involved in a singleton? That any elements in the set (if at all) would have certain properties makes (6) a universal assertion. Yet because the set has one element only, this also makes (6) a statement about a particular object. Hence, to correctly capture these duo relations, we need to translate (6a) as containing *two* categorical propositions, one universal and one particular, as in (6b).

Example (6): Socrates is wise.

(6a) There is exactly one object which is Socrates and for all object that is Socrates, that object is a wise person.

(6b) All object that is Socrates is a wise person (A-proposition) and some object that is Socrates is a wise person (I-proposition).

This translation will affect how we draw the Venn diagram of a sentence containing a singular term and subsequently how we test the validity of arguments involving it. We shall discuss more of this when we come to those sections.

Spotlight: Russell's Theory of Description

The discussion of singular terms is connected to Bertrand Russell's famous Theory of Description. Russell (1905) thought sentences such as 'the present king of France is bald' should be analysed as making three claims:

The present king of France is bald' is true if and only if:

- **a** There is a present king of France;
- **b** There is exactly one present king of France;
- **c** Whoever is a present king of France is bald.

(a) is the existence claim; (b) is the uniqueness claim; (c) is the attributive claim. Together they represent what a definite description asserts. In the case above, since in fact there is no present king of France, (a) is false. That makes the whole sentence false because one of its claims is not satisfied.

Alternatively, there is another way to represent singular terms: we take them as special cases and simply leave them as they are, i.e. we do not translate them into the four standard categorical forms. We do not treat a singular term as any set of objects, but simply represent it in the Venn diagram by a small letter to indicate it is a particular. This makes validity testing using a Venn diagram easier. We shall illustrate this in more detail after we have looked at the method.

▸ Negatives and double-negatives

Sometimes a sentence may express a negative meaning though its grammatical form is not exactly in the format of the E- or O-propositions. Alternatively, an affirmation can be achieved with double-negatives. In any of these cases, we should interpret the meaning in context and decide the quality and quantity of the relevant proposition.

> Example (7): All students are not stupid.
>
> No students are stupid.
>
> Example (8): No customers are not satisfied.
>
> All customers are satisfied.

'All students are not stupid' is not a standard categorical proposition. However, the word 'all' suggests a universal claim and the word 'not' expresses a negative assertion. Hence, the proposition expressed is actually an E-proposition, namely 'no students are stupid'. Likewise, the word 'no' in (8) suggests universality. The use of double negatives, 'no' and 'not', actually has the effect of affirmation rather than negation. Hence, the proposition expressed by (8) is universal and affirmative, so it is an A-proposition. In general, we can come up with the following translation rules.

> All S is not P = No S is P
>
> No S is not P = All S is P

Sometimes a negative is applied in the front of the sentence, meaning to negate the whole sentence rather than just one part

of it. To deal with these cases, we need to consider changing both the quality and quantity of the sentence. For example:

Example (9): Not everyone is wise.

'Everyone is wise' is universal and affirmative, an A-proposition. Under what condition would this proposition be false? Yes, when there is at least one unwise person! So one particular case can refute a universal proposition and we just need to find the opposite quality of the proposition, too. The negation of (9) is therefore 'someone is not wise', a particular and negative proposition. The negation of an A-proposition is thus an O-proposition.

(9a) Someone is not wise.

Indeed, there is a general rule. *In negating a whole proposition, we change both the quality and the quantity of the proposition.* The A-proposition is universal and affirmative. We change to particular and negative, so it is an O-proposition. Conversely, the negation of an O-proposition is an A-proposition. Likewise, an E-proposition (universal and negative) has its negation as an I-proposition (particular and affirmative), and vice versa. Indeed, the relations between A-O propositions and E-I propositions are called **contradictories** (a technical term which will be explained in Section 4.2). For example:

Example (10): It is not the case that accidents are impossible.

(10a) Some accident is possible.

'Accidents are impossible' can be interpreted as 'no accident is possible'. So, (10) requires a negation of the E-proposition. There only needs one accident to happen in order to refute the claim that no accident is possible. Hence, the negation of this E-proposition is an I-proposition. (10a) is thus the translation of (10).

In general, we can gather another rule of translation.

'It is false that...': A $\leftrightarrow$ O , E $\leftrightarrow$ I

Double negatives are treated likewise; we only need to negate or find the contradictories twice.

> Example (11): It is not the case that all politicians are not sincere.
>
> It is not the case that (no politicians are sincere).
>
> Some politicians are sincere.

'All politicians are not sincere' should be translated as 'no politicians are sincere'. This is because the word 'all' indicates that it is a universal assertion; however, the 'not' suggests that the quality is negative. Therefore, it is an E-proposition (universal negative), even though it uses the word 'all' as if it is an A-proposition. To negate an E-proposition is to find its contradictory, which is an O-proposition. Hence, the final translation is 'some politicians are sincere'.

The discussion of negating whole sentences opens up a whole area about immediate inference relations between propositions having the same subject and predicate. This actually goes beyond translation but touches on the logical relations among propositions – we explore this topic more in Section 4.2.

For now, let us summarize certain rules of thumb in doing translations as follows, and try an exercise. The key is always to interpret meaning in context and to be cautious about the quality and quantity of the proposition asserted by the sentence.

Key idea: Translation – categorical logic

Adjectives: translate to the corresponding set of objects.

Verbs: translate to the corresponding set of objects.

Alternative quantifiers:

every, any = all

none = no

most, a few, several, the majority/minority of = some.

Missing quantifier: understand in context and add the appropriate quantifier.

'Only': Only S is P = All P is S

Parameters (e.g. whenever, wherever, whatever, whoever): add sets of objects like time, occasion, events, things, people...

Singular term: there are two ways to deal with it – either translate it as bearing two propositions (one universal and one particular) or denote it by a letter as an individual, rather than by a circle as a set of things.

Negatives:

All A is not B = No A is B

No A is not B = All A is B

'It is false that...': A-proposition ↔ O-proposition, E-proposition ↔ I-proposition

Exercise 4.1: Translating into categorical propositions

Translate the following sentences into categorical propositions.

1. Orchids are not fragrant.
2. Not everyone worth meeting is worth having as a friend.
3. Happy indeed is she who knows her own limitations.
4. Only members can use the front door.
5. If he is asked to say a few words, he talks for hours.
6. None of my students is failing.
7. If you live in the dorms, you can't own a car.
8. The only tests George fails are the ones he takes.
9. You have nothing to lose but your chains.
10. Unless you pass this test you won't pass the course.
11. Most home movies are boring.
12. Not every part of Michael Jackson's face is original.

4.2 Immediate inferences and the traditional square of opposition

Categorical logic identifies the quality and quantity features of a proposition. We can anticipate different propositions with the same pair of subject and predicate in different relations. We call such sets of propositions **corresponding propositions.** Intuitively,

corresponding propositions are correlated in truth-values, too. Take for example, if it is true that all oranges are sweet, it would be intuitively true that some oranges are sweet. If it is false that all oranges are sweet, then it would be intuitively true that some oranges are not sweet. Such relations are called **immediate inferences**. Aristotelian logicians developed an elaborate system to understand these relations and formulate them in what is called the **traditional square of opposition**.

There are several logical relations featured in the traditional square of opposition, as follows.

CONTRADICTORIES

Contradictory is the relation between propositions with exactly opposite truth-values. It happens when propositions are opposite in both quality and quantity. If one proposition is true, its contradictory must be false and *vice versa*. That is, contradictories cannot have the same truth-value. It is understood that the contradictory of an A-proposition is an O-proposition, whereas the contradictory of an E-proposition is an I-proposition, and *vice versa*. Take the example of oranges. If all oranges are sweet (i.e. (12A) is true), then it would not be true that some oranges are not sweet (i.e. (12O) is false). For (12A) implies universality, so no exception would be found among oranges not to be affirmed of the property of sweetness. If (12A) is false, then not all oranges are sweet. This implies that some oranges are not sweet and (12O) is true.

(12A) All oranges are sweet.

(12E) No oranges are sweet.

(12I) Some oranges are sweet.

(12O) Some oranges are not sweet.

Similarly, if (12E) is true, that is no oranges are sweet, then it is impossible to find some oranges being sweet, so (12I) is false. If (12E) is false, that is, not all oranges are not sweet, it implies that some oranges are sweet, i.e. (12I) is true. We can easily work out what happens in other directions, say, if (12O) is true/false, then (12A) is false/true; and if (12I) is true/false, then 12(E) is false/true.

CONTRARIES AND SUBCONTRARIES

Contrary is the relation between two propositions which cannot both be true but can both be false. **Subcontrary** is the relation between two propositions which can both be true but cannot both be false. In categorical logic, these two sets of relations are formed by opposing the quality but not the quantity. Universal propositions A and E cannot be both be true but can be both be false; they are contraries. Particular propositions I and O can both be true but cannot both be false; they are subcontraries.

It is rather easy to see how these two relations are the case. A-propositions and E-propositions both describe exhaustively all objects involved in the subject term, yet the former asserts the subject term of a certain property but the latter denies it exactly. An individual cannot both have and have not the same property. Hence, the full set comprising such individuals also cannot both have and have not the property. Thus, A-propositions and E-propositions cannot be true together but can be false together (when no universality is achieved).

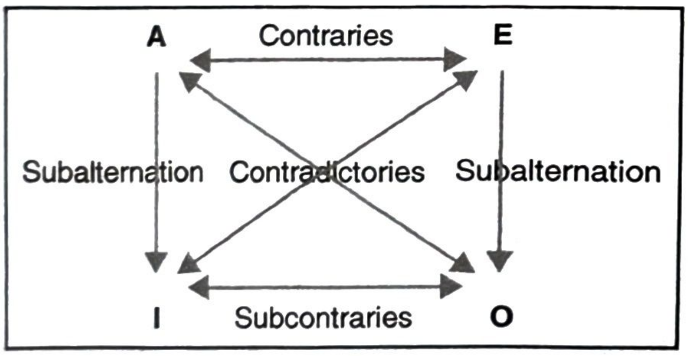
When particular objects are concerned, it can be that some individuals have the property while others have not. Thus, I-propositions and O-propositions can both be true. However, it cannot be the case that an individual both has and has not the property. Yet if an I-proposition is false, that means its corresponding E-proposition is true; if an O-proposition is false, the A-proposition is true. When I-propositions and O-propositions are both false, that means the E- and the A-propositions are both true. But A- and E-propositions cannot both be true since they are contraries. That entails I- and O-propositions cannot both be false. We can easily vindicate the case by repeating the orange example above.

SUBALTERNATION

A final set of relations is subalternation. A proposition P is in a **subalternation** relation with Q if whenever P is true, Q is true, though not *vice versa*, i.e. it is not the case that whenever Q is true, P is also true. This pair of relations occurs between A- and I-propositions, and between E- and O-propositions. It is generated from opposing quantity but not quality.

Universal implies particular. The whole is made up of its parts. If the whole is asserted or denied, of course some parts of it must be asserted or denied. So, it is intuitive that if an A-proposition is true, then its corresponding I-proposition is also true. Alternatively, if an E-proposition is true, then the corresponding O-proposition is true. Note that this relation applies only from universal to particular propositions and, unlike all other relations mentioned in this section, it does not hold in reverse order. It is because if some parts are asserted or denied, it does not guarantee that the whole is in the same condition. The set of objects may be mixed with individuals, some having a certain property while others not, thus no universality is achieved.

We have now identified all elements in the square of opposition. Putting the four types of propositions in each corner, we can present all possible relations among corresponding categorical propositions when some feature(s) of a proposition is (are) opposed. The result is a square as follows, duly called the traditional square of opposition.



OTHER TYPES OF IMMEDIATE INFERENCES

Apart from the immediate inferences mentioned above (contradictory, contrary and subcontrary, and subalternation), there are indeed other types traditionally espoused by Aristotelian logicians. These include the relations of conversion, obversion and contraposition. We may see some of these formats of sentences in daily life but not too often. More importantly, many of these relations are obsolete after the debate of existential import and subsequent widespread

adoption of the Venn diagram representation. We shall explain these issues in Section 4.3. For these reasons we shall not go into details about these relations but just state them for information; interested readers may refer to other logic books such as Irving Copi and Carl Cohen's *Introduction to Logic* (2010).

Conversion is an operation to form propositions by interchanging the subject and the predicate of an original proposition. Logicians studied the operation on each categorical proposition and came up with a table of valid conversions as follows.

▶ Table of conversions

A: All S is P	I: Some P is S (by limitation)
E: No S is P	E: No P is S
I: Some S is P	I: Some P is S
O: Some S is not P	(Conversion is not valid)

For example:

> Example (13): 'All whales are mammals' implies that 'Some mammals are whales'.

However, the conversion of the A-proposition is valid only if there are indeed whales. If the subject term does not denote any existent objects, then it seems absurd to assert any object denoted by the predicate term are objects denoted by the subject term (the latter being non-existent). For example:

> Example (14): 'All hobbits are hairy-legged creatures' does *not* imply that ' some hairy-legged creatures are hobbits'.

We cannot find among hairy-legged creatures any hobbits simply because there are in fact no hobbits! The conversion of A-propositions is therefore only valid by limitation. The validity for E- and I-propositions, however, is intuitive and general. Yet, conversion is not valid for an O-proposition because O-propositions do not exhaust members in the set of the predicate term P (the technical term for this is that P is not *distributed* in

O-propositions; the concept of distribution will be discussed in Section 4.6). Therefore, we cannot infer members of P to have any properties simply from how some S is not one of them.

Obversion is an operation to form propositions by changing the quality of a categorical proposition and replacing the predicate term by its complement (namely, changing P into non-P). For example:

> Example (15): 'All residents are voters' implies that 'No residents are non-voters'.

This is an example of obversion. 'All residents are voters' is affirmative. In obversion, it is changed to negative, so an E-proposition is used instead. Besides, P is also changed into non-P. This is a valid inference because if all residents can vote, that would mean no residents cannot vote, i.e. no residents are non-voters.

Obversions work according to the following table.

▶ Table of obversions

A: All S is P	E: No S is non-P
E: No S is P	A: All S is non-P
I: Some S is P	O: Some S is not non-P
O: Some S is not P	I: Some S is non-P

Finally, **contraposition** is a combined operation of obversions and conversions. It obverts a proposition, converts it, and then obverts it again. The resultant proposition takes the form of having non-P in the subject position and non-S in the predicate position. For example:

> Example (16): 'All scientists are serious people' implies that 'All non-serious people are non-scientists'.

Intuitively, (16) seems valid (and it does!) because if all scientists are serious, then that means we would not be able to find any scientists among non-serious people. Hence, all non-serious people are surely non-scientists.

Of course, we may doubt whether indeed, all scientists are serious. This does not hamper the validity of the argument though because validity is merely about what follows if the premise is true (re-read Chapter 1 if you are not sure about the concept of validity).

Contrapositions work according to the following pattern:

▶ Table of contrapositions

A: All S is P	A: All non-P is non-S
E: No S is P	O: Some non-P is not non-S (by limitation)
I: Some S is P	(Contraposition is not valid)
O: Some S is not P	O: Some non-P is not non-S

The properties of contrapositioning E- and I-propositions follow from the properties of converting A- and O-propositions, which are formed by first obverting the original E- and I-propositions. We can work out the proof procedures of contrapositions step by step following the tables of valid obversions and conversions. However, it seems not worth the effort from our present perspective.

4.3 Existential import

Although traditional inferences and operations seem elaborate, the traditional square of opposition and many of the immediate inferences have become obsolete after consideration of the issue of existential import. Modern logicians have abandoned every type of immediate inferences except contradictories and obversions. The consideration runs as follows.

A proposition has **existential import** if it is typically used to assert the existence of some objects. For example, 'My glass is filled with water' typically asserts that something, namely, my glass and water, exist and that some water is in my glass. So the proposition has existential import. Whereas, 'there are no mermaids' does

not have existential import because it denies the existence of some kinds of objects, namely, mermaids. The issue of existential import for categorical logic is whether each categorical proposition has existential import. It seems clear that particular propositions (I- and O-propositions) have existential import, though it is not clear whether universal propositions have it.

Intuitively, particular propositions have existential import because the following assertions seem self-contradictory.

> Example (17): Some apples are sour, but there are indeed no apples in the world.

> Example (18): Some trains are not steam-engineered, but there are indeed no trains.

If there are indeed no apples in the world, it would be absurd to assert some apples are sour. Similarly, if there are no trains, then we cannot assert trains being steam-engineered or not. Thus, to have asserted that some objects are so and so, we are logically *committed* to assert the existence of such objects as well.

However, it is not clear whether universal propositions have existential import. The following assertions do not seem as absurd as (17) and (18).

> Example (19): All trespassers are prosecuted, but there are indeed no trespassers.

> Example (20): Nothing falling into a black hole leaves it, whether or not anything ever does.

> Example (21): If Vulcan is the planet causing the disturbances in the orbit of Mercury, and any planet causing the disturbances in the orbit of Mercury will be at location L at 10.00 pm, then Vulcan will be at location L at 10.00 pm.

> Example (22): [Supposedly] Pegasus can fly, but there is in fact no Pegasus.

All the above involve use of **empty names**. Empty names do not only appear in fiction, they are also used in serious discourses such as law and science. In (19), 'trespassers' is not in general an empty name, but it can be in certain contexts when indeed

no one trespasses. Yet even when there are no trespassers, 'all trespassers are prosecuted' makes perfect sense and effects full linguistic and legal forces. Legal documents typically contain terms that do not have an actual instance yet. However, in case things happen as depicted, certain legal consequences follow.

(20) and (21) represent the use of empty names as theoretical terms. Scientists often explain phenomena by hypothesizing the existence of some unknown entities. Black holes are hypothesized to explain the death of stars; Vulcan is hypothesized to explain disturbances in the orbit of Mercury. It requires extremely complex processes to prove or disprove the existence of such entities. As far as we know, much scientific evidence supports the existence of black holes, yet it is now commonly rejected that Vulcan exists. My point is that sentences such as (20) and (21) make very good sense, even though the existence of the subject terms is not assured. They are both universal propositions. (20) is an E-proposition. (21) involves the sentence 'any planet causing the disturbances in the orbit of Mercury will be at location L at 10.00 pm' which can be formulated as an A-proposition. So it seems universal propositions are not committed to the assertion of the existence of objects they are about.

(22) is an example from fiction or mythology. A name may or may not be known to be empty; nonetheless we can utter something about it or even make intuitively true assertions about it based on its defining properties, such as Pegasus can fly.

From the observations above, we conclude that in general:

- Particular propositions (I- and O-propositions) have existential import.
- Universal propositions (A- and E-propositions) do not have existential import.

The existence of the objects represented by the subject term is *presupposed* in particular propositions. Yet it is not presupposed in universal propositions. It is as if a condition is attached to all universal propositions: All S is P, if there is any S; No S is P, if there is any S. Categorical logic does not handle conditional

sentences (the if-clauses). 'All S is P' is not to be understood positively as asserting that there are S and all S is P, but rather negatively as *denying any S which is not P*. In other words, it is possible that there is no S; yet if there is any S, then it is P. Thus, the A-proposition amounts to asserting the impossibility of finding an S outside of the set of P. Similarly, 'No S is P' is not to be understood as asserting that there is S and it is not P, but simply as asserting the *impossibility of getting an S which is P*.

These understandings are captured in a Venn diagram, which will be introduced in the next section. For now, let us explore how these understandings overturn the traditional square of opposition and immediate inferences.

IMPACT ON THE TRADITIONAL SQUARE OF OPPOSITION

Given the understanding that universal propositions do not have existential import, contraries, subcontraries, and subalternation have to go. Two propositions are contraries if they cannot be true together. However, A- and E-propositions can be true together now under the new interpretation. 'All S is P' means it is not possible to have an S that is not P. 'No S is P' means it is not possible to have an S that is P. When there is no S, then there is no S that is P, and there is no S that is not P. Hence, both impossibilities hold and the two propositions stand at the same time.

Two propositions are subcontraries if they cannot be false together. Yet I- and O-propositions are both false when there is no S. If there is no S, then there is no S that is P and there is no S that is not P. It just is nothing at all. So I- and O-propositions are no more subcontraries.

Subalternation is a relation of implication. According to the square of opposition, if an A-proposition is true, then its corresponding I-proposition is true. Similarly, if an E-proposition is true, its corresponding O-proposition is true. However, the subalternation relation holds only if A- and E-propositions have existential import. I- and O-propositions certainly have existential import. Given something cannot be derived from nothing, for I- and O-propositions to be true, the corresponding A- and E-propositions must entail the existence

of S, which is not the case under the new understanding. Therefore, the subalternation relation collapses.

IMPACT ON OTHER IMMEDIATE INFERENCES

The validity of obversion holds for all propositions but those of conversion and contraposition are further damaged by the new understanding of existential import. 'All S is P' no longer entails 'Some P is S' because universal propositions do not assert the existence of S and if there is no S, there will not be some P which is S. 'No S is P' also does not entail 'Some non-P is non-S' because the contraposition involves converting 'All S is non-P' to 'Some non-P is S', which is no longer valid for the reason just stated. Conversion of O-propositions and contraposition of I-propositions is already invalid from the start. The whole business of immediate inferences thus seems so truncated that it is no longer useful. We may just as well forget the complicated rules and tables and stick to the Venn diagram method.

To conclude, *the only valid immediate inferences left are contradictories and obversions.*

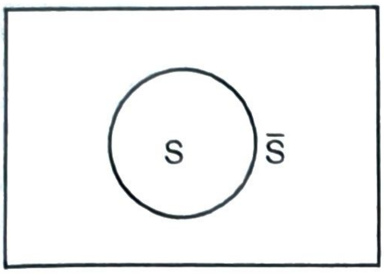
4.4 Venn diagrams

It is now a standard practice to represent categorical propositions using a Venn diagram, named after an English mathematician John Venn (1834–1923). This representation captures the modern interpretation of the existential import of categorical propositions. It also acts as a useful tool for proving validity of arguments in categorical logic.

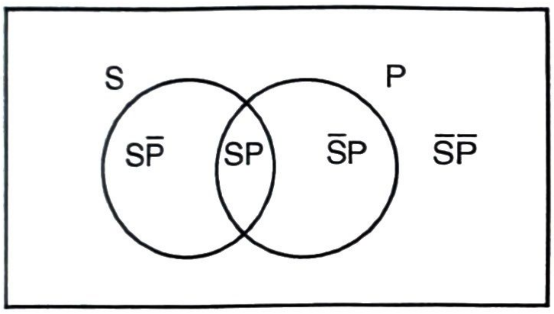
Categorical logic examines the relation between sets of objects. Venn diagrams take in that idea. Each set of objects is represented by a closed circle. Circles may overlap and they are suitably positioned to show all overlapping possibilities. The result is an exhaustive crafting of the whole logical space into partitions depending on the number of sets involved.

Let me illustrate. In logic, a domain is the universe or sphere to which the relevant logic system applies. When there is only one set of objects S in a domain, the universe is crafted into two areas: objects either belong to (fall within) S, or do not belong

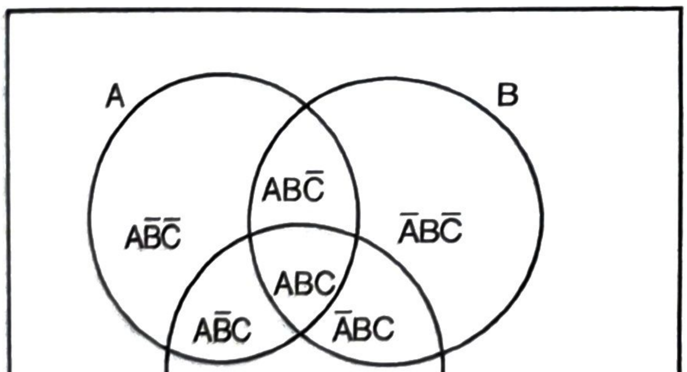
to (are situated outside of) it. The area inside is marked S, the outside non-S, or not-S, represented as $\overline{S}$.



When there are two sets of objects S and P, there are four areas of combinations in which an object may fall: it may belong to S but not P, S and P, P but not S, or neither S nor P. The logical space is crafted into four areas accordingly.



The more sets of objects, the more possible overlapping combinations there are. Indeed, the number of possible combinations is calculated to be 2^n, *n* being the number of sets in the domain. So, when there are three sets of objects A, B and C, there are eight possible combinations. The domain is then crafted into eight areas as follows.



We can use the Venn diagram to represent the four categorical propositions. Since each categorical proposition is about two sets of objects S and P, we need diagrams with four crafted areas. We also need some conventions of notation. Different books may adopt different conventions; we will adopt the following.

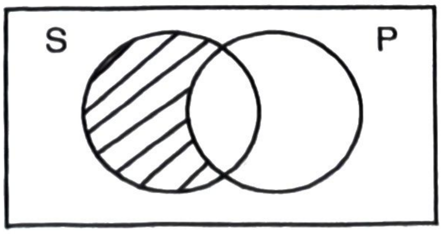
- Shading an area means there is no object in that area.
- The presence of a tick '✓' means there is *some* object in that area, understood in the technical sense as having at least one object.
- The presence of a question mark '?' means it is probable but not certain that this area has some object.
- Small letters such as 'a', 'b', 'c' represent the presence of particular individuals in that area.

According to the modern understanding, universal propositions do not have existential import. So universal propositions are represented not by ticks but by shading out of areas where there cannot be objects. The A-proposition asserts the impossibility of having an S that is not P. Whereas the E-proposition asserts the impossibility of having an S that is P. The two propositions are expressed as follows.

A: All S is P

= Nothing is S but not P

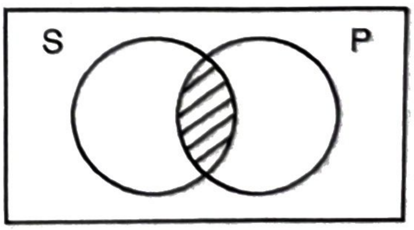
= shade away area that represents $S\overline{P}$



E: No S is P

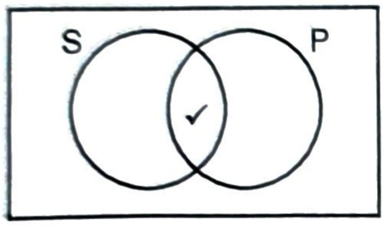
= Nothing is both S and P

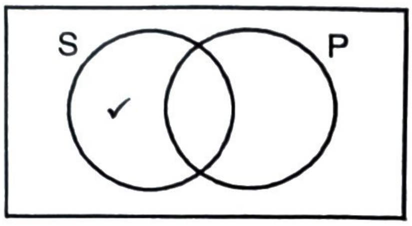
= shade away area that represents SP



Particular propositions have existential import. Thus, we can use ticks to express that there are objects asserted in relevant areas.

I: Some S is P	O: Some S is not P
= Something is both S and P (area SP)	= Something is S and not P (area $S\overline{P}$)





Now we can also see graphically how A- and O-propositions and E- and I-propositions are contradictories. They write about the same area and assert exactly the opposite of the other. A- and O-propositions assert the absence and the presence of objects in the area $S\overline{P}$ respectively; while E- and I-propositions assert the absence and the presence of objects in the area SP respectively.

Some conversions hold. The diagram of the E-proposition (the E-diagram, for short) can be read as 'No S is P' as well as 'No P is S'. The I-diagram can be read as 'Some S is P' as well as 'Some P is S'. However, conversions of A- and O-propositions do not hold. The A-diagram does not show 'All P is S' or 'Some P is S', for there could be no S at all. The A-proposition only precludes S from being not P, it does not assert the existence of any S. Conversion is invalid for the O-proposition because there is no tick in P whatsoever, so it is not possible to infer any properties of P from the O-diagram.

Obversions hold and the Venn diagrams show them. The A-diagram shows 'No S is non-P'. The E-diagram shows 'All S is non-P' because it shows the impossibility for S to be P. So if there is S, S must reside in the non-P area. In a similar vein, the I-diagram can read 'Some S is not non-P' and the O-diagram 'Some S is non-P'.

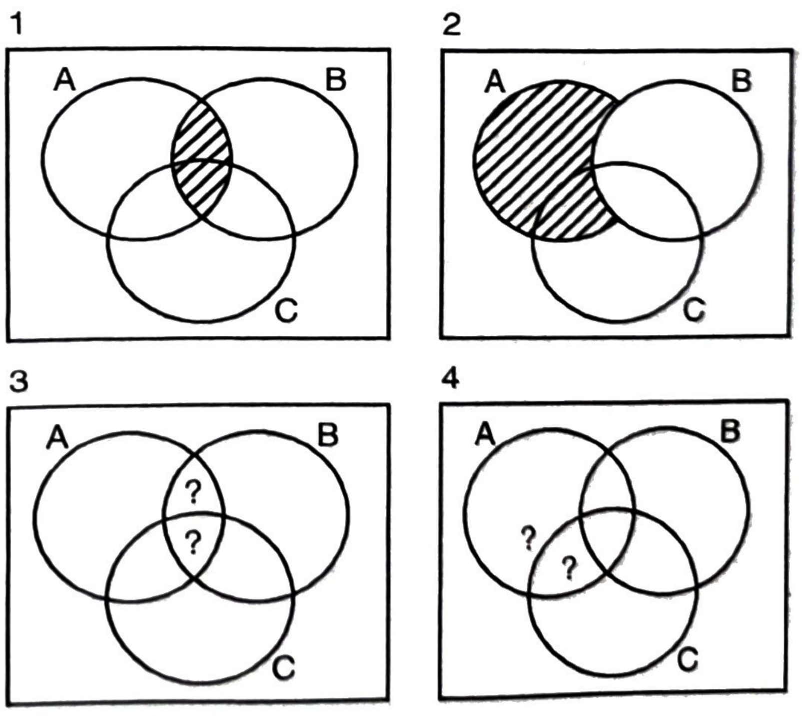
Contraposition is valid for A- and O-propositions only and these can be read from respective Venn diagrams, too. The A-diagram shows 'All non-P is non-S' because there is no non-P that is S. The O-diagram shows 'Some non-P is not non-S' as some non-P is S really. However, contrapositions for E-proposition and I-proposition do not hold, for the same reasons that conversions for A- and O-propositions do not hold respectively.

The Venn diagram is a handy tool. It is intuitive and visual, literally laying open any possibilities for things to be. It indicates the existential import of various types of categorical propositions. It also encompasses all results of valid immediate inferences, if any, such that there is no need to take any steps of inference in the mind or memorize the traditional square of opposition and any rules or tables anymore.

Now let's do an exercise to build up our familiarity with Venn diagrams.

Exercise 4.2: Using Venn diagrams to represent categorical propositions

What categorical propositions do these diagrams show?



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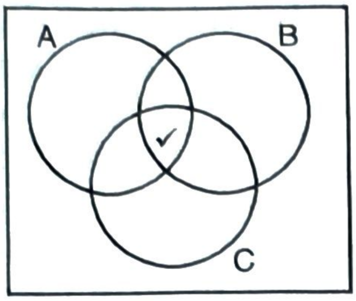


4.5 Using Venn diagrams to test the validity of arguments

CATEGORICAL SYLLOGISM AND ITS STANDARD FORM

It is now common to use Venn diagrams to test the validity of a typical type of argument in categorical logic called categorical syllogism. A **syllogism** is an argument that has exactly two premises and one conclusion and a **categorical syllogism** is a syllogism that comprises only categorical propositions. Since each categorical proposition concerns two sets of objects and an argument is supposed to relate all parts concerned, a categorical syllogism can only deal with exactly three sets of objects in total. Categorical syllogism is a popular argument type in daily life. Below is an example:

> Example (23): All gardeners are lovers of nature.
>
> Some lovers of nature are environmental activists.
>
> Therefore, some gardeners are environmental activists.

Traditionally, the subject of the conclusion is called **the minor term** (S) of the argument; the predicate of the conclusion is called the **major term** (P) of the argument. The term that appears only in the premises is called the **middle term (M)**. The premise containing the minor term is called the **minor premise**. The premise containing the major term is called the **major premise**.

A categorical syllogism is in its standard form if and only if it satisfies the following format:

1 Each proposition (premises and conclusion) is a categorical proposition.

2 The syllogism deals exactly with three terms.

3 The three terms are identified as the minor term, the major term and the middle term.

4 The argument is to be arranged in the following order:

 Major premise

 <u>Minor premise</u>

 Conclusion

For example, the major term of (23) is 'environmental activists', the minor term is 'gardeners', and the middle term is 'lovers of nature'. (23) appears in the following form:

 All S is M (minor premise)

 <u>Some M is P</u> (major premise)

 ∴ Some S is P (conclusion)

This is not in the right order, so it should be rearranged into the standard order as follows.

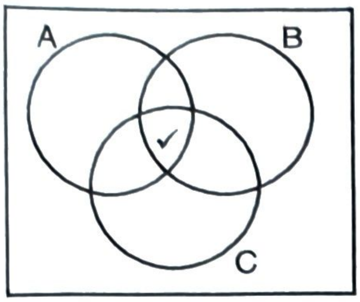
 Some M is P (major premise)

 <u>All S is M</u> (minor premise)

 ∴ Some S is P (conclusion)

Aristotelian logicians went on to represent the argument form of a categorical syllogism based on the standard forms. They used two more concepts: moods and figures. The **mood** of a syllogism presents the type of categorical proposition (A, E, I, O) in the order of the major premise, the minor premise and the conclusion. For instance, the mood of the syllogism in (23) is IAI.

5



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Aristotelian logicians went on to represent the argument form of a categorical syllogism based on the standard forms. They used two more concepts: moods and figures. The mood of a syllogism presents the type of categorical proposition (A, E, I, O) in the order of the major premise, the minor premise and the conclusion. For instance, the mood of the syllogism in (23) is IAI.

The **figure** indicates the positions of the middle term in the premises. There are four figures, as follows:

First figure	Second figure	Third figure	Fourth figure
M – P	P – M	M – P	P – M
S – M	S – M	M – S	M – S
∴ S – P	∴ S – P	∴ S – P	∴ S – P

(23) has the format of the first figure. Hence, the argument form of (23) can be fully represented as IAI-1.

Using moods and figures, we can identify all possible argument forms in categorical syllogism. There are altogether 256 possible forms, because there are only $4^3 = 64$ moods and each mood has four figures. If we are patient and industrious enough, we can test the validity of all argument forms exhaustively. There are not many valid argument forms, really. (I shall reveal how many there are by the end of this chapter. Make a guess now!)

It is necessary to identify the exact standard form of a categorical syllogism; otherwise the traditional method of test for validity, which is called the rule method (to be introduced in Section 4.6), cannot apply. However, the standard form is not significant when the Venn diagram method is used. The Venn diagram method only requires the identification of the conclusion and the premises and to formulate them in the form of a standard categorical proposition (A, E, I or O). The rest, including the identification of terms, moods and figures, are no longer important.

TESTING VALIDITY USING THE VENN DIAGRAM

The basic idea is to present all premises (about three sets of objects) in one diagram and literally *see* whether the conclusion is already contained in the premises, namely, whether the diagram shows the required conclusion. It is a valid test for validity because it fulfils what validity means. Drawing the premises on the diagram means to suppose all the premises are true. If it follows that the conclusion is contained, then that means if the premises are true, the conclusion is true. Since the Venn diagram is exhaustive and includes all possibilities,

this implies that the containment is also necessary. In the case where the premises contain the conclusion, it means that if the premises are true, the conclusion must be true. This satisfies the definition of validity exactly.

Key idea: The Venn diagram – step by step

1. Rewrite the argument in standard form using symbols A, B, C or S, P, M to represent the sets of objects concerned. As explained above, it does not matter in a Venn diagram which symbol represents which term as long as the notation is consistent. There is also no need to identify major and minor premises.
2. Draw each premise onto the Venn diagram. Now, one order of drawing is important in the Venn diagram method: *draw universal propositions (A and E) first* before particular propositions (I and O). This is because universal propositions are all about shading areas out; doing this first makes the remaining options more certain. We shall illustrate how this works out in the examples that follow.
3. Determine the areas related to the conclusion should it be true.
4. If the conclusion is contained in the presentation of the premises, then the argument is valid. Otherwise, it is invalid.
5. Be careful about '?'. It shows uncertainty as to whether there are definitely objects in that area. If the conclusion requires that the area definitely has something in it, then merely having a '?' is not sufficient to guarantee validity.

Let us start with some simple examples and work our way up to more complex ones.

Example (24): All men are vile men.

All handsome men are men.

Therefore, all handsome men are vile men.

The argument is already formulated in standard form with premises on top followed by the conclusion and each proposition is a standard categorical proposition. We only need to symbolize. Remember, it does not matter what each symbol stands for in this method as long as the symbols are used

consistently. However, I will honour the distinction of major and minor terms in the following presentations, to serve as examples to prepare those who want to learn the rule method later.

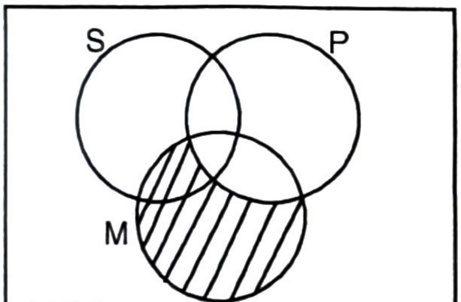
Let S = handsome men, P = vile men, M = men. The argument is represented in symbols:

All M is P

<u>All S is M</u>

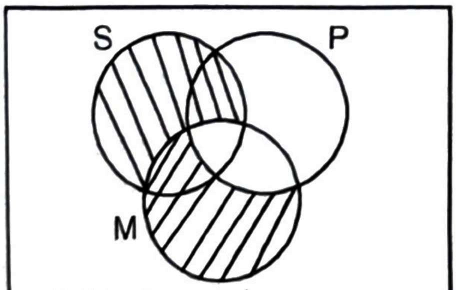
∴ All S is P

Draw the premises in a Venn diagram. There are three terms in the argument, so we draw a diagram containing three sets of objects. Three sets of objects have $2^3 = 8$ possible areas of combination. All premises are universal propositions; hence it, does not matter which one is drawn first. So we draw 'All M is P' as follows.



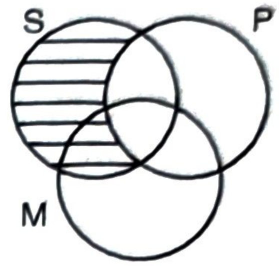
(24) first premise

We then add the second premise 'All S is M':



(24) both premises

If the conclusion 'All S is P' is true, areas representing $S\bar{P}\bar{M}$ and $S\bar{P}M$ should have no objects in them. That is, the following diagram should obtain:



(24) conclusion

Compare the diagrams of the two premises and the conclusion. In the former, $S\bar{P}\bar{M}$ is shaded due to the second premise and $S\bar{P}M$ is shaded due to the first premise. This is exactly what the conclusion requires. We conclude that the premises already contain the conclusion. The argument is thus valid.

We showed above every procedure step by step for illustrative purposes. Yet when we are more familiar with the method, we only need to draw the diagram which contains all premises, and then determine the validity of the argument and explain our reasons. There is no need to draw the conclusion. In a class setting, I would ask students to mark the areas to be covered by the conclusion in different ink. This should then clearly indicate that students know where to look in order to determine the validity of the argument.

Example (25): Whenever there is lightning, there is thunder.

Whenever there is thunder, there is rain.

Therefore, whenever there is lightning, there is rain.

This example may not look like (24), yet upon reformulating it into standard form, we can see that they actually share the same logical form. (24) is valid; hence, so is (25).

(25) can be formulated as follows. First, we turn all propositions into categorical propositions.

All occasions of lightning are occasions of thunder.

All occasions of thunder are occasions of rain.

Therefore, all occasions of lightning are occasions of rain.

We identify the minor term as 'occasions of lightning', the major term as 'occasions of rain'. The second premise is indeed the major premise and the first one the minor premise. The major premise should be stated first. So the position is rearranged.

All occasions of thunder are occasions of rain.

All occasions of lightning are occasions of thunder.

Therefore, all occasions of lightning are occasions of rain.

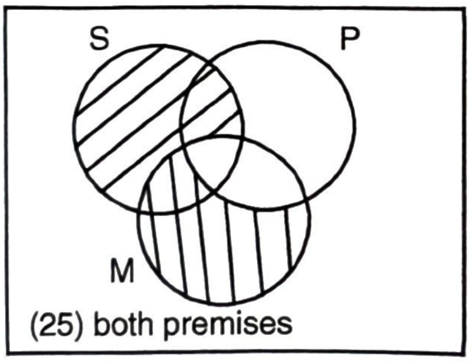
Let S = occasions of lightning, P = occasions of rain, M = occasions of thunder

All M is P

All S is M

∴ All S is P

It is now plain that (25) shares the same structure as (24). Draw the Venn diagram to prove its validity.



(25) both premises

The conclusion requires that both areas of $S\bar{P}\bar{M}$ and $S\bar{P}M$ have nothing in them. The premises satisfy these requirements. Hence, the premises contain the conclusion and the argument is valid. The diagram of (25) is exactly the same as that of (24). This is another illustration of the formal character of logic (See Section 2.1).

Example (26): Some black cats catch mice.

Any cats that catch mice are good cats.

Therefore, some black cats are good cats.

Deng Xiaoping, the Chinese leader from the late 1970s to the mid-1980s, famously said, 'It doesn't matter whether the cat is black or white, as long as it catches mice.' This showed his pragmatism. (26) contains a similar idea. Let us reformulate the argument into its standard form:

Some black cats are mice-catching cats.

All mice-catching cats are good cats.

Therefore, some black cats are good cats.

Since the minor term is 'black cats' and the major term is 'good cats', the first premise should be the minor premise and the second premise the major premise. Rearrange and symbolize.

All mice-catching cats are good cats.

Some black cats are mice-catching cats.

Therefore, some black cats are good cats.

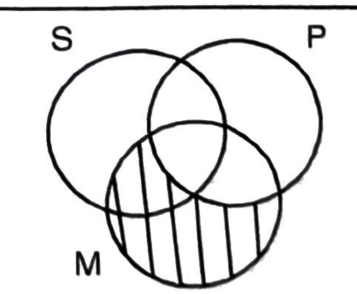
Let S = black cats, P = good cats, M = mice-catching cats

All M is P

Some S is M

∴ Some S is P

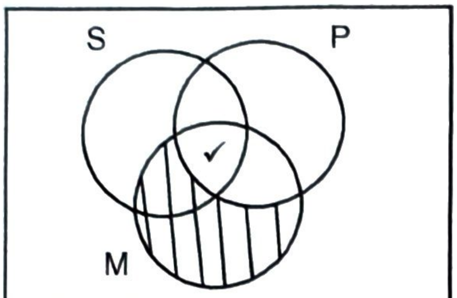
This example shows how the rule of drawing the universal propositions first is important. This rule applies irrespective of which premise(s) the universal proposition(s) is/are.



(26) universal premise

Now draw the particular proposition. 'Some S is M' requires that either $SP\bar{M}$ or SPM has objects. Yet since $S\bar{P}M$ is already

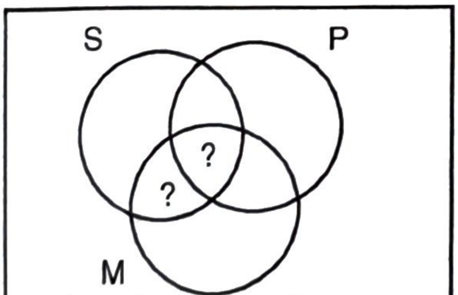
shaded out because of the universal premise, for the particular premise to be true, SPM must have objects. Since it is a certain case, we can put a tick with confidence, rather than a question mark, to the area of SPM.



(26) both premises

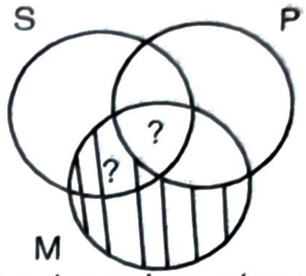
The conclusion is 'some S is P'. For it to be true, either $SP\overline{M}$ or SPM has to have objects. Now the premises make it the case that SPM has objects. This gives just what is needed for the conclusion to be true. So the premises contain the conclusion and the argument is valid.

If we do not observe the rule of drawing universal propositions first, we may draw the particular proposition first. But then since either SPM or $S\overline{P}M$ fits the requirement and nothing is certain, so we can only put question marks in both areas, as follows.



(26) particular premise

When we then draw the universal proposition, we only see question marks and it would mask the fact that indeed certainly some objects are in SPM.



(26) universal premise on top (wrong)

It is not the case that we can always use a tick. Sometimes even after drawing the universal proposition, there is still uncertainty about where objects would go. (27) is an example of this.

Example (27): Some college students are not scientists.

No scientists are illiterate.

Therefore, some college students are illiterate.

The minor term is 'college students' and the major term is 'illiterate'. So, the second premise is the major premise while the first is the minor premise. Rearrange in standard form and symbolize.

No scientists are illiterate.

<u>Some college students are not scientists.</u>

Therefore, some college students are illiterate.

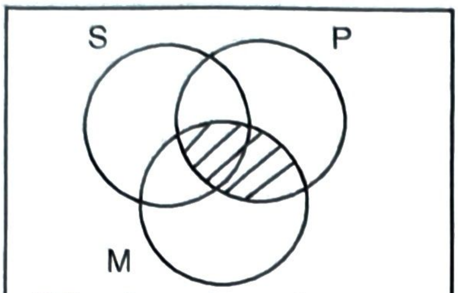
Let S = college students, P = illiterate, M = scientists

No M is P

<u>Some S is not M</u>

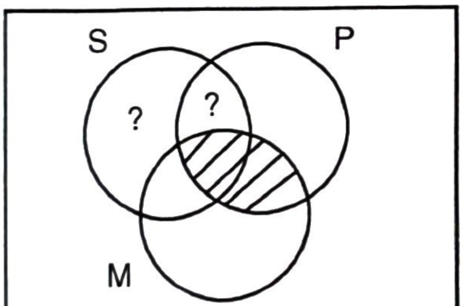
∴ Some S is P

Draw the universal premise first. 'No M is P' means there is nothing that is both M and P; so we should cross out the overlapping area between M and P.



(27) universal premise

Then draw the particular premise. 'Some S is not M' means there can be objects in $SP\bar{M}$ or $S\bar{P}\bar{M}$. No area is crossed out, so that means either or both areas may have objects. Logically, 'some' only means 'at least one'. When there is only one such object, it is not certain whether that object would fall in $SP\bar{M}$ or $S\bar{P}\bar{M}$. Therefore, we cannot determine which area has objects for sure. Therefore, we cannot put any ticks but can only put question marks in both possible areas.



(27) both premises

Now let us determine the validity. The conclusion 'some S is P', if obtained, would have something in the overlapping areas of S and P, namely, $SP\bar{M}$ or SPM. SPM is shaded out but $SP\bar{M}$ has a question mark, showing that it may have some objects in it. However, since it is only a possibility and nothing from the premises *guarantee* that it surely is the case, the premises do not ensure that $SP\bar{M}$ has objects, i.e. they do not ensure the conclusion follows. Validity guarantees that if the premises are true, the conclusion is true. (27) does not provide such a guarantee and thus is invalid.

Indeed, Venn diagrams of categorical syllogism showing question marks are seldom valid, exactly because of the uncertainties these question marks represent.

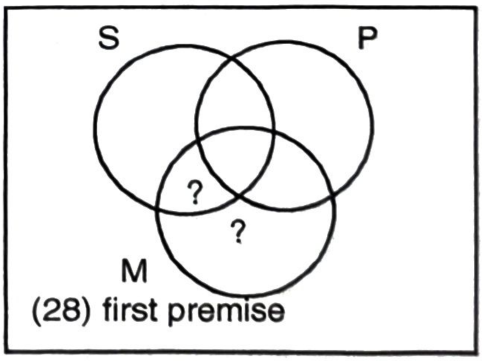
There can be more than two question marks present in a Venn diagram of a categorical syllogism. The following is an example of this.

> Example (28): Some diamonds are not gems.
>
> Some carbonates are diamonds.
>
> Therefore, some carbonates are not gems.

The minor term is 'carbonates' and the major term is 'gems'. The argument is in standard form. Let us proceed to symbolization.

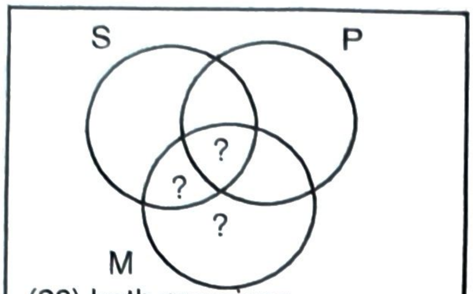
> Let S = carbonates, P = gems, M = diamonds
>
> Some M is not P
>
> <u>Some S is M</u>
>
> ∴ Some S is not P

Draw the diagram for the first premise 'Some M is not P'. It entails that objects exist in $S\bar{P}M$ or $\bar{S}\bar{P}M$, yet it is not certain which area must have objects. We thus put question marks in both areas.



(28) first premise

Draw the second premise. It requires some objects in SPM or $S\bar{P}M$, yet it does not specify which. So both areas should be marked by a question mark. $S\bar{P}M$ already gets a question mark from the first premise; there is no need to mark double. We just add one more question mark to SPM.



(28) both premises

The conclusion 'some S is not P' requires that $S\bar{P}\bar{M}$ or $S\bar{P}M$ has objects. Now, $S\bar{P}M$ has a question mark, so there can be objects. However, it is not certain that it must have objects, so the premises do not guarantee the truth of the conclusion. The argument is invalid.

We have gone through all the basics by now. You should begin to master the skills. Try the following exercise on your own. After this, we shall come back and discuss more complex cases.

Exercise 4.3: Testing validity using Venn diagrams

1 Test the validity of the following syllogism forms using Venn diagrams.
  - **i** All A is B. No B is C. Therefore, no A is C.
  - **ii** No A is B. Some B is C. Therefore, some C is not A.
  - **iii** Some A is B. Some B is C. Therefore, some A is C.
  - **iv** Some A is not B. All B is C. Therefore, some A is C.
  - **v** Some A is not B. All C is B. Therefore, some A is not C.

2 Identify the conclusion and premises for each argument below. Then rewrite the syllogisms in standard form using the symbols provided and determine their validity using Venn diagrams.
  - **i** No television stars are certified public accountants (C), but all certified public accountants are people of good business sense; it follows that no television stars (A) have good business sense (B).
  - **ii** All juvenile delinquents (C) are maladjusted individuals, and some juvenile delinquents are products of broken homes; hence some maladjusted individuals (A) are products of broken homes (B).

iii No intellectuals (A) are successful politicians (B), because no shy and retiring people (C) are successful politicians, and some intellectuals are shy and retiring.

iv Some Christians (A) are not Methodists (B), for some Christians are not Protestants (C), and some Protestants are not Methodists.

MORE EXAMPLES

We mentioned in Section 4.1 some skills that are used in translating ordinary language into categorical propositions. Let us now apply them in solving some more complex arguments.

▶ Rhetorical expressions

Example (29): All men are human, and some humans are French. So some men *have to be* French.

The conclusion of this argument is 'some men have to be French'; we identify it because of the conclusion indicator 'so'. The premises are 'all men are human' and 'some humans are French'. The conclusion is not a categorical proposition as it has the phrase 'have to be French', rather than simply 'is' or 'is not'. Linguistically, 'have to be' emphasizes the certainty of the assertion. However, logically, it does the same thing as to assert some men are French. The difference in tone does not matter to the content. Hence, we ignore the rhetoric and simply state the conclusion as an I-proposition.

All men are human beings.

<u>Some human beings are French.</u>

Therefore, some men are French.

Symbolize, let S = men, P = French, M = human beings.

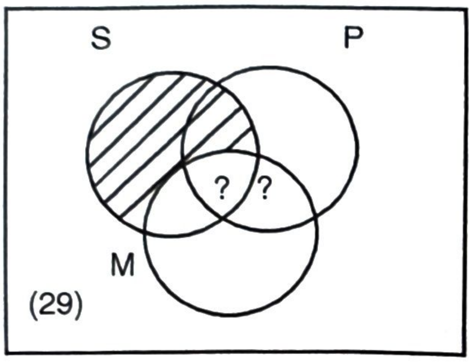
All S is M

<u>Some M is P</u>

∴ Some S is P

'Some M is P' is the major premise and should be placed as the first premise. 'All S is M' is the minor premise and should be

identified as the second premise. However, the order of major or minor premise does not matter in a Venn diagram. What matters is that we draw the universal premises before the particular ones. Hence, we draw 'All S is M' first. 'Some M is P' requires the existence of objects in SPM or $\bar{S}$PM. Since none of the areas is shaded by the universal premise, both areas possibly have objects. We put in question marks to indicate the uncertainty.



The conclusion 'some S is P' requires that either SP$\bar{M}$ or SPM has objects. SP$\bar{M}$ has no object because the area is shaded by the universal premise. However, it is still uncertain whether SPM has things in it because we only have a question mark. Thus, even if the premises are true, they do not guarantee the truth of the conclusion. The premises do not contain the conclusion. The argument is thus invalid.

▸ 'Only'

Example (30): All physicists are clever. It is because all physicists are scientists, and only scientists are clever.

First, we identify the conclusion and the premises. The conclusion of this argument is the first sentence. Beware that conclusions are not necessarily presented at the end but can be placed at the front or in the middle of a discourse. The premise indicator 'because' indicates what follows are the premises. We have the argument identified as follows.

All physicists are scientists.

<u>Only scientists are clever.</u>

Therefore, all physicists are clever.

The main issue is how to interpret the second premise which contains the word 'only'. Does 'only scientists are clever' mean 'all scientists are clever people' or 'all clever people are scientists'? In Section 4.1, we discussed that 'Only A is B' is equivalent to 'All B is A'. 'Only A is B' does not assert all A to be B; it just says that being A is necessary for being B. That means, if something is B, it must also be A. So, 'only scientists are clever' should mean 'all clever people are scientists', not 'all scientists are clever people'. Now we can formulate the argument in standard form and symbolize as follows.

All physicists are scientists.

<u>All clever people are scientists.</u>

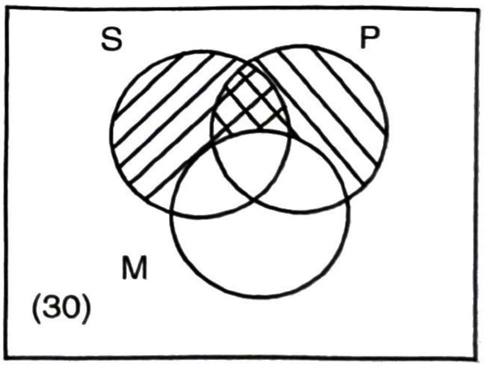
Therefore, all physicists are clever people.

Let S = physicists, P = clever people, M = scientists

All S is M

<u>All P is M</u>

∴ All S is P



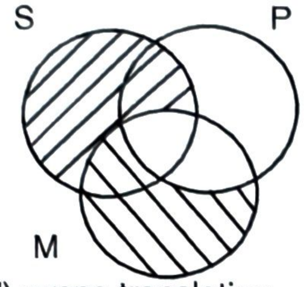
The conclusion 'all S is P' requires that $S\bar{P}\bar{M}$ and $S\bar{P}M$ are both shaded. Now $S\bar{P}M$ is not shaded by the premises. Hence, the premises do not contain the conclusion. The argument is invalid.

Note that it is a completely different argument if we translate the premise 'only scientists are clever' wrongly into 'all scientists are clever people'. The argument form and its Venn diagram for that argument (30') would be as follows.

All S is M

<u>All M is P</u>

∴ All S is P



(30') wrong translation

Here, $S\bar{P}\bar{M}$ and $S\bar{P}M$ are both shaded. Therefore, the premises contain the conclusion. The argument is valid!

The moral is that we must be very careful when translating propositions. If we do it incorrectly, the whole proof will be wrong in the sense that it won't be the proof for the original argument at all.

▶ Singular terms

Example (31): No logician is entirely sane. Tom is a logician. Tom is not entirely sane.

As discussed before, singular terms imply both universality and particularity, i.e. they do imply that some object falls into the set and whatever falls into the set has the properties ascribed. Hence, singular terms can be treated in two different ways.

The simpler way is to take a singular term as presenting a particular object. It is not an indefinite object, so it should not be represented by an arbitrary tick or question mark, but by a definite letter *a, b, c* to indicate that it is a certain definite object. Under this treatment, (31) can be formulated and tested in the following way:

No logician is an entirely sane person.

<u>Tom is a logician.</u>

Therefore, Tom is not an entirely sane person.

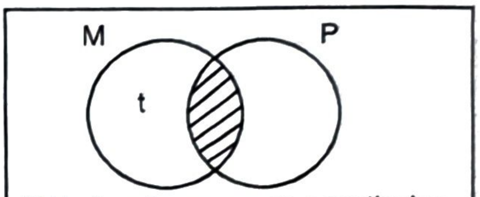
Let t = Tom, P = entirely sane persons, M = logician

No M is P

t is M

∴ t is not P

There are only two sets of objects, P and M. Here, Tom is not represented as a set of objects but a particular object t. The Venn diagram thus contains two circles only.



(31) singular name as a particular

Draw the universal proposition first. The proposition is 'No M is P', so we shade out the overlapping area of M and P. The particular object t belongs to M, according to the second premise. Since MP is shaded out, t cannot be in there. t must then be situated in $M\overline{P}$. We thus put a letter t in that area to indicate that the definite object is in there.

The conclusion requires that t is in $M\overline{P}$ or $\overline{M}\overline{P}$. Now t is in $M\overline{P}$. So the premises contain the conclusion and the argument is valid.

Alternatively, a singular term can be treated as prescribing a singleton set – a set that contains exactly one element. Each proposition containing a singular term has to be analysed as containing two categorical propositions: one universal and one particular. The whole argument thus becomes:

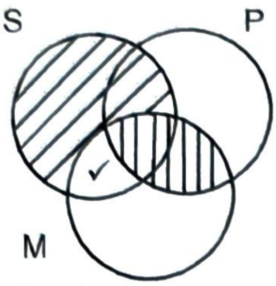
Let S = any object that is Tom, P = entirely sane persons, M = logician

No M is P

All S is M and some S is M

∴ No S is P and some S is not P

To test the validity of the argument, we need to draw both parts of the premise containing the singular term and also see whether the premises contain every part of the conclusion.



(31) singular term as a set

To draw 'No M is P', we shade out the overlapping areas between M and P. Then we draw 'All S is M', shading out $S\bar{P}\bar{M}$ and $SP\bar{M}$. 'Some S is M' requires that either $S\bar{P}M$ or SPM has objects. SPM is already shaded out by the first premise. Hence, only $S\bar{P}M$ possibly has objects. We are certain about this and hence can put a tick, rather than a question mark, in that area.

To determine validity, the conclusion requires that it is not possible for anything to be both S and P. This is fulfilled by the premises as both $SP\bar{M}$ and SPM are shaded out. The conclusion also requires that there is some S which is not P, so either $S\bar{P}\bar{M}$ or $S\bar{P}M$ has objects. Now $S\bar{P}M$ does have objects (it has a tick in it). So the second part of the conclusion is also fulfilled. Therefore, the premises do contain the conclusion, and the argument is valid.

Both kinds of treatment of singular terms are conceptually sound. We may adopt either. Just be reminded that if you treat singular terms as singleton sets then you must make sure all parts of the propositions are expressed and taken care of.

▶ Parameters

Parameters can be handled quite straightforwardly, as in the following example.

> Example (32): Wherever there is fire, there is smoke. There is no smoke in the basement. Hence, there is no fire in the basement.

We add the parameter 'places' or 'locations'. Thus, the argument can be translated as follows:

All locations of fire are locations of smoke.

No locations in the basement are locations of smoke.

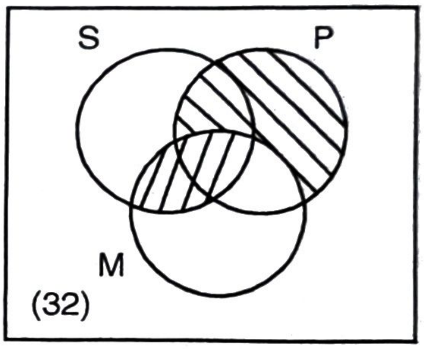
Therefore, no locations in the basement are locations of fire.

Let S = locations in the basement, P = locations of fire, M = locations of smoke

All P is M

No S is M

∴ No S is P



The conclusion requires that $SP\bar{M}$ and SPM are both shaded out. They are indeed shaded out by the premises. Hence, the premises contain the conclusion and the argument is valid.

▶ Negatives

Let us try a more complicated example that involves parameters and negatives.

> Example (33): Whenever I'm in trouble, I pray. And since I'm always in trouble, there is not a day I don't pray.

The conclusion, 'there is not a day I don't pray', is a universal proposition with double negatives. It means 'no days are days I don't pray', so the format is 'No S is not P'. We discussed in 4.1 that a double negative is equivalent to affirmation. So the proposition becomes 'I pray every day', or 'all days are days that I pray' (an A-proposition).

The premises of (33) are 'whenever I'm in trouble I pray' and 'I'm always in trouble'. The terms 'whenever' and 'day' suggest

adding a suitable parameter related to time. We may try 'times' or 'days'; in either case we would need to be consistent and interpret other propositions in the syllogism using the same word. We may use any the following:

All the times I am in trouble are the times that I pray.

All times are the times I am in trouble.

Therefore, all times are the times that I pray

Or:

All the days I am in trouble are the days I pray.

All days are the days I am in trouble.

Therefore, all days are the days I pray

We may also rewrite the last formulation, for easier reading, as:

All my trouble days are my praying days.

All days are my trouble days.

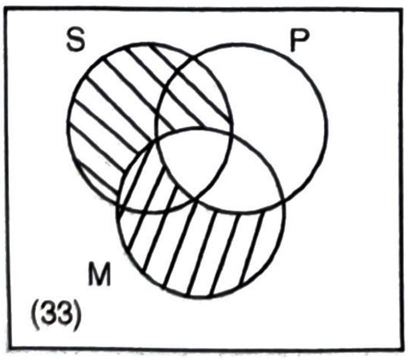
Therefore, all days are my praying days.

Let S = days, P = my praying days, M = my trouble days

All M is P

All S is M

∴ All S is P



The conclusion requires that $S\bar{P}\bar{M}$ and $S\bar{P}M$ have no objects in them. The areas are shaded by the premises. Therefore, the premises contain the conclusion and the argument is valid.

▶ Identifying three terms

Categorical syllogism deals with exactly three terms. Any arguments with more than three terms cannot be dealt with. In those cases, we need to adjust the terms artificially to fit into the requirement. Our final example is an example of this and it also encompasses other aforesaid issues such as negatives.

> Example (34): Not all unemployed are sensible drinkers. Only debtors are alcoholics. Therefore, it is false that no unemployed are in debt.

We first identify the conclusion and the premises as follows.

> Not all unemployed are sensible drinkers.
>
> <u>Only debtors are alcoholics.</u>
>
> Therefore, it is false that no unemployed are in debt.

The minor term is the unemployed; the major term is the people in debt. Hence the first premise is the minor premise and the second the major premise. Rearrange the order.

> Only debtors are alcoholics.
>
> <u>Not all unemployed are sensible drinkers.</u>
>
> Therefore, it is false that no unemployed are in debt.

Reformulate the propositions into standard categorical forms. The major premise involves the word 'only'. 'Only A is B' is equivalent to 'All B is A'. So this premise becomes 'All alcoholics are debtors'. The minor premise presents a negated A-proposition, in the form of Not-(all S is P). The contradictory of an A-proposition is an O-proposition. Hence, we change this premise to 'Some unemployed are not sensible drinkers'. The conclusion involves a negation, too. The contradictory of an E-proposition is an I-proposition. So the conclusion becomes 'Some unemployed are in debt'. The whole argument is now in the mood of AOI:

> All alcoholics are debtors.
>
> <u>Some unemployed are not sensible drinkers.</u>
>
> Therefore, some unemployed are people in debt.

However, the argument still involves five terms: 'the unemployed', 'sensible drinkers', 'debtors', 'alcoholics', and 'people in debt', to enumerate. To reduce them to exactly three terms, we look for synonyms and antonyms. 'Debtors' are synonymous with 'people in debt', so they can be treated as the same term. Sensible drinkers and alcoholics are close enough as opposites. Even though they are not exact antonyms, at least alcoholics are not sensible drinkers. So let us treat them as complementary to each other in reference. The argument becomes:

All alcoholics are debtors.

<u>Some unemployed are alcoholics.</u>

Therefore, some unemployed are debtors.

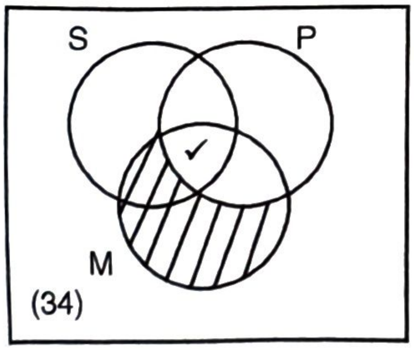
The argument is now in clean, neat, standard categorical forms. It is an AOI-1 argument. We can now proceed to symbolization and validity testing.

Let S = unemployed, P = debtors, M = alcoholics

All M is P

<u>Some S is M</u>

∴ Some S is P



It is a valid argument.

By far, the last example is the most complicated because each of its propositions requires some reformulation. We also see now that once an argument is 'cleaned up' to the standard forms, the remaining procedures, such as symbolization and validity testing, are easy.

To sum up, the Venn diagram method of testing validity is intuitive. It is justified as a proof to validity because the diagram exhausts all possibilities of how things are. If it shows that a conclusion is already contained in some premises, then the conclusion must be true if the premises are true, which simply satisfies the definition of validity. The Venn diagram carries correct understanding of existential import. It does not require memorization of substantial rules about the content of arguments but understanding of general concepts, such as validity and construction rules of the diagram. There is no need to identify the major, minor, and middle terms or to arrange the premises in strict order.

Exercise 4.4: More validity tests

Identify the premises and conclusion in each question. Rewrite the syllogisms in standard form and determine their validity using Venn diagrams. Remember to set your symbols and be careful with the translations! Some questions are quite complex.

1. Because no weaklings are true liberals, and all labour leaders are true liberals; so, no weaklings are labour leaders.
2. All centaurs are mammals, for some mammals are not horses, and no horses are centaurs.
3. All criminal actions are wicked deeds. All prosecutions for murder are criminal actions. Therefore, all prosecutions for murder are wicked deeds.
4. Some reformers are fanatics, so some idealists are fanatics, since all reformers are idealists.
5. All men are mortal. Socrates is a man. Therefore, Socrates is mortal.
6. 'She told me that she had a very simple attitude toward her students which was in fact no different from her feelings about people in general. That was, all her life she'd spoken only to people who were ladies and gentlemen. Since none of the students of 9D were ladies and gentlemen, she never spoke to them, never had, and never would.' (James Herndon (1968), *The Way It Spozed to Be*)

7 It is not true that less destructive nuclear weapons are not dangerous, because less destructive nuclear weapons make it easier for a nuclear war to begin and any nuclear war is dangerous.

8 Whenever there's a challenge there's opportunity, so we are grateful whenever there is a challenge, because we are grateful whenever there is opportunity.

4.6 The rule method

The Venn diagram method is a good method and is the one most in use currently. However, there are other ways to test validity. The rule method, for instance, was a popular method before the Venn diagram was invented. The rules had been altered to reflect the correct understanding of existential import. It is a fast method in that it only requires determination according to a few rules.[7] However, it requires proper formatting of arguments in standard forms, memorization of the rules, and employment of another concept called distribution. The justification for the rule method is less formal and obvious, too. Either way, both methods are fine and good for use in daily reasoning. My recommendation is that readers master the Venn diagram method, and treat the rule method as an optional extra.

DISTRIBUTION

To learn the rule method, we must first understand the concept of **distribution**. A term is distributed if every member of the class it refers to is *exhausted* by the proposition containing it. What terms are distributed in each of the four categorical propositions? Let us mark all distributed terms in a categorical proposition with a superscript ^{D}. Some authors may use other notations, such as circling or underlining.

A-proposition: All S^D is P

E-proposition: No S^D is P^D

I-proposition: Some S is P

O-proposition: Some S is not P^D

The subject term of an A-proposition is distributed because the proposition says about every S that it is also a P. However, the predicate term of an A-proposition is not distributed. Take an example. 'All mammals are vertebrates' says of every mammal having the property of being a vertebrate. It does not, however, say of all vertebrates what property they have. A vertebrate can be a mammal or not a mammal, tall or short, big or small, the proposition does not specify or characterize any common properties of them.

Both the subject and the predicate terms of an E-proposition are distributed. It is because the proposition asserts a property for every member of S, namely, that it is not a member of P. And it asserts a property for every member of P, too, which is that it is not a member of S. Take the example, 'No soap is acidic.' If this proposition is true, then not only can we not find any soap among acidic objects, we also cannot find any acidic objects among soap. Thus, we know something about every soap and something about every acidic object.

The I-proposition does not have any distributed terms. Consider 'Some flowers are red'. It says nothing exclusively about flowers nor red things. Some flowers may be not red and some red things are other than flowers. Hence, the proposition does not exhaust any set S or P.

Finally, the predicate term but not the subject term of an O-proposition is distributed. It may need more effort to figure out why though. The subject term is not distributed because, after all, the O-proposition is a particular proposition. It asserts properties for some members of S but not all of them. For example, 'some oranges are not sweet' tells us about oranges that some of them are not sweet while some may be. However, surprisingly, it does tell us of some property of all members of P, namely, that there is something that is not among any of them. For example, to ascertain 'some oranges are not sweet', one needs to go through all sweet objects to find out that those particular oranges are not among them. So the proposition does exhaust all sweet things.

THE RULES

We can now state the rules of categorical syllogism. A categorical syllogism is valid if and only if it satisfies all the following rules:

1 The number of negative claims in the premises must be the same as that in the conclusion.

2 The middle term must be distributed at least once.

3 Any term that is distributed in the conclusion must be distributed in its premises.

4 A particular conclusion must not be derived from universal premises alone.

Because there can only be one conclusion in an argument, the first rule implies that the number of negative claims in the premises can only be zero or one, not two. Any syllogism with two negative premises is invalid. When the conclusion is affirmative, both premises have to be affirmative. When the conclusion is negative, then there can be exactly one negative premise, but not two, for the argument to be valid.

Let us give some intuitive explanation of the rules. These are not formal proofs, however, they just attempt to make sense of the rules.

Recall what categorical syllogism does in general. A categorical syllogism is an argument that links the minor term to the major term via the middle term. The middle term acts like a bridge to connect two sets of things. Affirmation is the action to connect two sets of things while negation tears them apart. An argument is valid only if the conclusion is true *as a result of* the effect of the bridge, namely, that the conclusion is true, not just by itself, but is true *because of* the connections offered by the middle term. The connections to the middle term make sure that the conclusion is not obtained by accident. Validity is all about guarantee and certainty.

When both premises are negative, neither the minor term nor the major term are connected to the middle term. In such a case, there can hardly be a linkage established between the minor term and the major term by the presence of the middle term. Therefore, the argument containing two negative premises cannot be valid.

When the conclusion is affirmative, some connection is established between the minor term and the major term. Yet for a syllogism to be valid, the connection must be established by virtue of the middle term. Hence, the minor term and the major term must both be connected to it. The result is that both premises have to be affirmative in order to make the connection.

In the final case, when the conclusion is negative, one of the terms (minor or major) must be connected to the middle term while the other is not. The connection is necessary because validity is all about guaranteeing results. On the one hand, if none of the minor and the major terms are connected to the middle term, then the connection established will not be a result of the argument, but perhaps an accident. Logic looks for certainty, not accidents. Accidental connection means invalidity. On the other hand, if both premises are affirmative, then the conclusion cannot be negative, as the second case mentioned above. Nothing is there to tear apart the minor or major terms from the middle term; hence, the conclusion would not have a negation. In sum, a valid syllogism with a negative conclusion must have exactly one negative proposition as the premise.

The second rule focuses on the function of the middle term. As mentioned above, the middle term works like a bridge. If a bridge does not extend and reach out to all members it connects, then the connection may fail. For instance, members of S may connect to some members of M while members of P may connect to other members of M such that eventually no relation is established throughout between all members of S or P. It is like a broken bridge. To prevent this from happening, at least one side (S or P) must be connected to the whole bridge, which is M. Distribution of M ensures this will happen. In other words, the second rule ensures that the middle term covers entirely S or P, or both, so S and P won't be just two loose ends on the bridge without proper connection at all.

The third rule is tasked to make sure the argument delivers what it claims. If the conclusion claims something about all

members of a set, then at least all members of that set must be involved in the connection process. Hence, if a term is distributed in the conclusion, it must also be distributed in the premise. It does not matter whether that term is the minor term or the major term.

According to modern interpretation, universal propositions have no existential import while particular propositions have existential import. However, none of the above rules precludes inference of particular claims from purely universal claims. For instance, if the conclusion is an I-proposition, neither the major nor the minor term is distributed. So it does not require them to be distributed in the premises either. All that is needed is that the middle term be distributed at least once and that both premises are affirmative. Hence, in principle, AAI-1, AAI-3 and AAI-4 are valid, if only three rules apply. Similarly, some figures of AEO or EAO arguments may be valid, too. So it is still possible to infer something from nothing and commit the existential fallacy. The fourth rule exists to amend this situation.

HOW THE RULE METHOD WORKS

Follow the steps below when confronted with a categorical syllogism:

- First, rewrite the syllogism strictly in the standard form. This includes: distinguishing the minor term, the major term and the middle term, identifying the major premise and minor premise and putting them in the right order.
- Second, identify the distribution of terms, using notations such as underlining, circling, or adding a D in superscript.
- Finally, check whether any of the rules are violated. If so, the argument is invalid. If not, it is valid.

▶ Examples

For the sake of brevity, the following examples are already formulated in the standard form.

Example (35): Some pedigrees are expensive dogs.

No adoptable dogs are pedigrees.

Therefore, some adoptable dogs are expensive dogs.

Let S = adoptable dogs, P = expensive dogs, M = pedigrees

Some M is P

No S^D is M^D

∴ Some S is P.

This is an IEI-1argument. It *violates the first rule.* The conclusion is affirmative; hence, according to the first rule, there should be no negative premise. The minor premise, however, is negative. Hence, it is an invalid argument. It does not violate the second rule because the middle term 'pedigrees' is distributed once in the premises. It does not violate the third rule because none of the terms in the conclusion is distributed. Hence, it does not matter either whether they are distributed in the premises. Finally, it does not violate the fourth rule because there is a particular proposition in the premise.

Example (36): Some keyboard-players are not percussionists.

All pianists are keyboard-players.

Therefore, some pianists are not percussionists.

Let S = pianists, P = percussionists, M = keyboard players

Some M is not P^D

All S^D is M

∴ Some S is not P^D

This is an OAO-1 argument. It does not violate the first rule. The conclusion is negative and there is exactly one negative premise. However, the middle term 'keyboard-players' is not distributed in the major premise or the minor premise. The argument thus *violates the second rule*. It does not violate the third rule, though. The minor term 'pianists' is not distributed in the conclusion, so it does not matter whether it is distributed in

the minor premise. The major term 'percussionists' is distributed in the conclusion, yet it is also distributed in the major premise; hence, there is no problem at all. It also does not violate the fourth rule because although the conclusion is particular, the major premise is particular, too.

Example (37): No developers are land owners.

All developers are creditors.

Therefore, no creditors are land owners.

Let S = creditors, P = land owners, M = developers

No M^D is P^D

<u>All M^D is S</u>

∴ No S^D is P^D

This is an EAE-3 argument. It does not violate the first rule. The conclusion is negative and there is exactly one negative premise. The example does not violate the second rule because the middle term 'developers' is distributed at least once in the premises. However, this example *violates the third rule.* The minor term 'creditors' is distributed in the conclusion but it is not distributed in the minor premise, though the major term is fine. The fourth rule does not apply because the conclusion is not a particular proposition.

Example (38): All creatures with hairy feet are creatures that like hot summers.

All hobbits are creatures with hairy feet.

Therefore, some hobbits are creatures that like hot summers.

Let S = hobbits, P = creatures that like hot summers, M = creatures with hairy feet.

All M^D is P

<u>All S^D is M</u>

∴ Some S is P

This is an AAI-1 argument. It does not violate the first rule. There is no negative conclusion and no negative premises either. It does not violate the second rule because the middle term 'creatures with hairy feet' is distributed in the major premise. It does not violate the third rule either because no terms in the conclusion are distributed so it does not matter whether they are distributed in the premises. It only *violates the fourth rule.* The conclusion is a particular proposition but all premises are universal. This fallacy is evident in that there are in fact no hobbits; hence, we cannot infer that some hobbits actually are so and so.

It is possible that a syllogism has violated more than one rule. As long as a rule is violated, the argument is invalid.

In introducing the standard form of categorical syllogism, we mentioned that there are not many valid argument forms out of the 256 possible ones. I can tell you now that there are indeed only 15 valid forms of categorical syllogism, listed in the Spotlight box below! It would be troublesome to test all 256 forms using Venn diagrams. However, with the rule method, we can eliminate invalid argument forms fairly quickly and easily. Interested readers may want to rehearse the process themselves. It is actually quite fun and rewarding – you literally exhaust all possible valid arguments in minutes!

Spotlight: Only 15 valid argument forms of categorical syllogism!

AAA-1	EAE-1	IAI-3	OAO-3
AEE-2	EAE-2	IAI-4	
AEE-4	EIO-1		
AII-1	EIO-2		
AII-3	EIO-3		
AOO-2	EIO-4		

Exercise 4.5: Testing validity using the rule method

Rewrite the following argument forms in standard forms. Identify the distributed terms. Then determine their validity by explaining which rules are broken, if any.

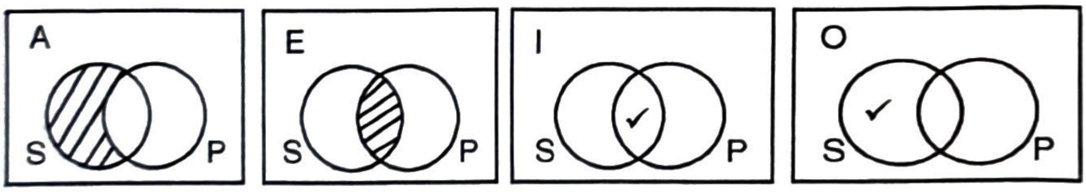
1 All A is B. All A is C. Therefore, all B is C.
2 Some A is not C. Some C is not B. Therefore, some A is not B.
3 No B is C. All A is B. Therefore, some C is not A.
4 All B is C. No A is C. Therefore, no B is A.
5 Some A is C. All B is C. Therefore, some A is B.
6 All A is C. All B is C. Therefore, some A is B.
7 All C is B. Some A is C. Therefore, no A is B.
8 All C is A. All B is A. Therefore, some C is not B.

4.7 Chapter summary

Let us sum up what we have learned so far.

MAIN POINTS OF CATEGORICAL LOGIC

- There are four types of categorical propositions: A E I O
- A Venn diagram lays out *all possibilities* of how things are.
- Be familiar with the Venn diagrams of each proposition type.



- Understand the notations: shade, tick, question marks, small letters, and which area stands for what property of things
- Know how to use Venn diagrams to *test the validity* of a categorical syllogism
- Identify the conclusion and the premises.

- Translate each sentence into a standard categorical proposition.
- Identify the minor term, the major term and the middle term.
- Arrange the propositions in the following order: the major premise, the minor premise, the conclusion.
- Draw the diagrams of the premises. First the universal premises if any, then the particular premises if any.
- Determine validity by observing whether the premises contain the conclusion *definitely*.

TRANSLATION NOTES

- **'Only':** Only A is B = All B is A
- **Negatives:** All A is not B = No A is B; No A is not B = All A is B
- **Negatives:** Not all A is B = Some A is not B; it is false that no A is B = Some A is B
- **Logically unimportant information:** e.g. 'got to be' = 'is'
- **Singular terms:** represented by small letters and they work like a '✓'
- **Parameters:** add parameters to translate the terms when needed

THE VENN DIAGRAM METHOD:

- Draw a diagram with three circles dividing the space into eight areas.
- Draw universal premises before particular premises.
- Be careful about ticks and question marks.
- Determine validity. If the premises already contain the conclusion, then the argument is valid. Otherwise, it is invalid.

THE RULE METHOD:

An argument is valid if and only if it does not violate any of the following rules.

- The number of negative claims in the premises must be the same as the number of negative claims in the conclusion.
- The middle term must be distributed at least once.
- Any term that is distributed in the conclusion must be distributed in its premises.
- A particular conclusion must not be derived from two universal premises.

5

Propositional logic

In this chapter you will learn about:

- ***the quest for completeness***
- ***truth-tables and five logical connectives***
- ***computing truth-tables***
- ***translating ordinary language***
- ***testing validity***
- ***the short truth-table method***
- ***natural deduction***

> *'I know what you're thinking about,' said Tweedledum: 'but it isn't so, nohow.'*
> *'Contrariwise,' continued Tweedledee, 'if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic.'*
> Lewis Carroll (1871), *Through the Looking Glass*, Chapter 4

5.1 The quest for completeness

This chapter introduces another logical system called propositional logic.

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Propositional logic is variably called symbolic logic, sentential logic, propositional calculus, or truth-functional logic. While categorical logic analyses the *internal* structure (subject–predicate relation) of a proposition and focuses on the relations among sets of objects, propositional logic, in contrast, takes a proposition as the basic unit and focuses on the logical *relations between* them. This is why the above quote from Lewis Carroll's novel characterizes logic dramatically as all about negation, conjunctions and conditionals.

A **proposition** is a logical structure that is capable of being true or false. A symbol in propositional logic, usually represented by a small letter in italics, stands for an atomic proposition. An **atomic proposition** is the basic unit that expresses a complete thought. Examples include 'snow is white', 'Socrates is wise', etc. Atomic propositions combine to build up **complex propositions**. For instance, the complex proposition 'snow is white and the sky is blue' is made up of atomic propositions 'snow is white' and 'the sky is blue'; similarly, 'If Socrates is wise, then he should escape from the prison' is composed of 'Socrates is wise' and 'Socrates should escape from the prison'. Typically, propositional logic deals with arguments like the following:

> Example (1): If things go well, I will join you soon.
>
> Things go well.
>
> Therefore, I will join you soon.

Example (2): Either you like it, or you will do something about it.

You don't like it.

Therefore, you will do something about it.

\+ A proposition is different from a sentence. A 'proposition' is a logical notion. It signifies an abstract entity, the content of which can be true or false. However, a sentence is a linguistic item. We have discussed their differences in Section 2.3. To recap:

- Different sentences may express the same proposition, e.g. 'snow is white' and '*la neige est blanche*'.
- The same sentence may express different propositions, such as indexicals, demonstratives, and context-sensitive expressions (e.g. 'I am hungry', 'it is hot').
- Sentences may be incomplete, such as 'Fire!' Yet a proposition must express a complete thought capable of being true or false.

Propositional logic uses propositions as the basic unit. Propositions are either true or false. We call truth and falsity
\+ the truth-values of a proposition. Classical logic (including all three logic systems introduced in this book) accepts the principle of **bivalence**, i.e. there are only two possible values given to a proposition, true or false. Some logic systems accept other values; for example, intuitionist logic accepts 'indeterminate' propositions, free logic accepts 'truth-valuelessness', fuzzy logic accepts various degrees of truth. These logic systems are called **many-valued logics**.

Although different propositions express different thoughts, what matters in logic (regardless of the acceptance of the principle
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of bivalence) is only their truth-values, or more accurately, their various possible **truth-value assignments**. A truth-value assignment is the truth-value assigned to a complex proposition depending on the truth-values of its constituent atomic propositions. Truth-values matter; indeed, reasoning is all about truth-preservation. Validity is defined as whether a conclusion must be true in case the relevant premises are true. Propositional

logic tracks how the truth-value of a proposition (the conclusion) varies in case other propositions (the premises) are true or false.

Spotlight: Why do only truth-values matter? Frege's idea of reference

We have already introduced Frege's distinction of sense and reference in Section 2.3. Frege proposed that the reference of a name is the object picked out by the name and the reference of a proposition is its truth-value. Following this view, all true propositions have the same referent: *Truth*. We think that different propositions express different truths only because they express different thoughts. A thought is the sense of a proposition. So, '2 + 5 = 7', 'the Earth orbits the Sun', 'God is benevolent' (if it is true) all stand for the same truth, though they express different thoughts. This may sound a bit weird, but it paves the way to abstract propositions into just a handful of values to operate. Logic can then become a purely formal matter and leave the complicated and highly varied issues of meaning behind.

To present all that a proposition can be, we draw a truth-table showing all possible assignments of truth-values of a proposition. A **truth-table** is a *complete* list of all possible truth-value assignments of a proposition. Completeness is significant because only by exhausting all possibilities can there be any guarantee of certainty in reasoning. We do not want to misjudge an argument as valid when indeed a conclusion may be false if the premises are true.

5.2 Truth-tables and five logical connectives

An atomic proposition can only be true or false. So the truth-table of any atomic proposition p is presented as follows:

p
T
F

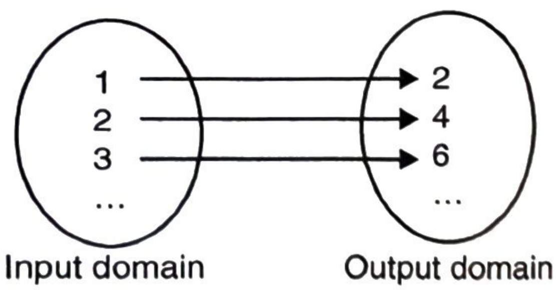
When there are several atomic propositions, we can expect more possible combinations of truth-values. For example, if there are two propositions, the following truth-table exhausts all possible combinations of their truth-value assignments.

p	*q*
T	T
T	F
F	T
F	F

Each row represents a possible truth-value assignment. So the above table means: Concerning two propositions p and q, it is possible that p is true and q is true, or p is true and q is false, or p is false and q is true, or both p and q are false.

Indeed, the number of total possibilities can be calculated by the formula of 2^n, n being the number of atomic propositions. Thus, when one atomic proposition is concerned, $n = 1$ and there are $2^1 = 2$ possible truth-value assignments. When two atomic propositions are involved, $n = 2$ and there are $2^2 = 4$ possible assignments. When there are three atomic propositions, $n = 3$ there are $2^3 = 8$ possibilities. When there are four atomic propositions, there are 16 possibilities, and so on.

Propositional logic is interested in the relations between propositions. In particular, it is interested in a type of relation called truth-functional relations. A **function**, in the sense the term is used in mathematics and logic, is an operation that maps a certain object to another. Thus, for example, the function '× 2' when applied to positive integers yields the following sets:



A **truth-function** is a function that maps objects according to their truth-values. In other words, that an object is an element

of the output domain is determined solely by the truth-value of an object in the input domain and the respective function. A **truth-functional relation** is thus a relation in which the truth-value of the combined sentence is *completely determined* by the truth-values of the component sentences.

Not all relations between propositions are truth-functional. Consider the following daily examples:

Example (3): It is warm and sunny.

Example (4): Mary has cancer because she smokes.

'It is warm and sunny' is composed of two atomic propositions 'it is warm' and 'it is sunny', linked by the word 'and' in a connection called conjunction. 'It is warm and sunny' is true only if the weather is indeed warm and the weather is indeed sunny. However, it is not true if the weather is not warm, or if the weather is not sunny, or if the weather is neither warm nor sunny. Hence, the truth-values of the constituent sentences *completely determine* the truth-value of the complex sentence. The relation of conjunction is therefore truth-functional.

Contrast this with the relation of 'because', as in (4). It is possible that Mary has cancer and Mary is a smoker, yet Mary does not get cancer because she smokes. Her cancer may be caused by other things, such as exposure to radioactive materials or hereditary reasons. Indeed, scientists have yet to give a complete explanation of how cancer develops. Even if statistically many smokers do get cancer, it is still possible that some smokers never get it. Therefore, the truth of the constituent propositions does not completely determine the truth of the complex proposition in this case. Causation is therefore not a truth-functional relation.

Logicians identify *five common truth-functional relations*, represented by five logical connectives in various symbols. Different sets of notations are in use, depending on convention. It does not matter which set of notation is used as long as consistency is observed.

Because these relations are truth-functional, they are defined by truth-tables, i.e. all respective possible truth-value assignments under their operation. In what follows, we draw their respective

truth-tables. Then we look for words and phrases in ordinary language that indicate such relations.

Spotlight: How many basic relations are there?

There is a debate about how many relations are necessary and basic in propositional logic. We introduce five connectives in this book. However, equivalence can be expressed as a conjunction of material implication and material implication can be expressed in terms of negation and disjunction. So it is argued that at least some of these connectives are not necessary in the logic system. Logicians even invented an artificial relation called the **Sheffer stroke**, represented as '|' or '↑', to stand for the negation of conjunction. Negation, disjunction, conjunction and subsequently implication and equivalence can all be represented using the Sheffer stroke (see also Spotlight box below). Therefore, the number of necessary symbols in a bivalent logic system can even come down to one! Similarly, there is another truth-functionally complete connective like the Sheffer stroke called the Peirce's arrow, represented as '⊥' or '↓'. All five common connectives can be expressed in terms of it, too. While this all seems neat, exactly because of the artificiality of the matter, it seems also unimportant for common sense reasoning to dwell on this issue. Perhaps the issue is simply pragmatic, that is, we adopt the connectives because they are commonly used in ordinary discourse, not because they are irreducible to other logical relations.

NEGATION (¬, ~)

Negation is the contradiction of the original proposition. If the original proposition is true, then its contradiction is false. If the original proposition is false, then its contradiction is true. Since a proposition can only be true or false, these two scenarios exhaust all the possibilities of a contradiction. The truth-table of negation is as follows.

p	$\neg p$
T	F
F	T

Simply put, the rule of negation is as follows:

> A proposition and its **negation** cannot be true or false at the same time.

Negation is often indicated by words or phrases in ordinary language like 'not', 'not that', 'it is not the case that', 'it is false that', etc. For example, the negation of (5) below is (5') and there are several ways to express such negation:

> Example (5): Snow is white.
>
> (5') Snow is *not* white.
>
> *It is not the case that* snow is white.
>
> It is false that snow is white.
>
> *Not that* snow is white.

Negation may not look like a relation, especially when we only add a 'not' inside a sentence, but it really is. It is a relation because the words or phrases have the function to pick out a proposition which is contradictory to the original proposition. So they do operate on a sentential level to take a proposition to another proposition, namely, its contradictory. We can see this clearly if phrases like 'it is not the case that' are used.

CONJUNCTION (&, ∧, ·)

Conjunction is a relation that joins constituent propositions together. It obtains when all its constituents obtain. Each constituent is called a conjunct. Conjunction must involve two conjuncts. Since conjunction always joins two propositions, a truth-table of conjunction must have 2^2, i.e. 4, rows of possible truth-value assignments. The truth-table of conjunction is as follows:

p	*q*	*p & q*
T	T	T
T	F	F
F	T	F
F	F	F

A conjunction is characterized as:

> A **conjunction** is true only when both components (conjuncts) are true.

Conjunction is typically indicated by words and phrases such as 'and', 'also', 'as well as', 'whereas', 'while', etc. Interestingly, phrases like 'but', 'nevertheless', 'though', 'in spite of', also indicate conjunction. Although a sentence containing these phrases involves contrasts, the sentence is true only if all its constituents are true. The tone or linguistic convention of usage does not matter in logic. Hence, the following are all examples of conjunction.

> Example (6): David and Patrick are hard working.

> Example (7): To pass this course, one has to hand in all assignments as well as to pass the examination.

> Example (8): Time will fade but our love will never end.

> Example (9): The collective is strong even though individuals are weak.

Although the word 'and' in (6) apparently only joins two names, the sentence is indeed a conjunction of two propositions, namely, 'David is hard working' and 'Patrick is hard working'. So the function is a sentential one. (7), (8) and (9) are true only if all constituent propositions are true. Hence, they are still conjunctions regardless of the words used to connect them.

DISJUNCTION (v)

Disjunction obtains when one of the elements obtains and each element is called a disjunct. Disjunction is indicated in ordinary language by words such as 'or', 'either... or'. The following are typical examples:

> Example (10): Students *or* staff are welcome to the party.

Although the word 'or' seems to apply only to the subjects, the sentence is actually composed of two simple sentences. So (10) is equivalent to (10'):

> (10') Students are welcome to the party *or* staff are welcome to the party.

For (10) to be true, a party attendant has to be a student or a staff member. A person only needs to satisfy one of the two requirements in order to be welcome to the party. That is to say, the satisfaction of one true disjunct is sufficient to make the whole proposition true. However, if both disjuncts are true, would the disjunction be true as well? Intuition says yes, and so it is. If a person is both a student and a member of staff (say, a staff member enrols on a part-time course at the same institution, or a student gets a part-time job working on campus, etc.), that person is surely welcome to the party. In fact, he or she would be doubly qualified to attend. This illustrates why in general disjunction is true when both disjuncts are true. In fact, the only situation in which a disjunction is false is when there is no true disjunct at all. We can state the truth-table of disjunction as follows.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction is thus characterized in the following way:

> A **disjunction** is false only when both components (disjuncts) are false.

One may wonder, however, that in ordinary language, sometimes 'or' is used in a way that does not mean the above but to indicate that exactly one of the cases is true. That is, it does not accept the sentence as true when both disjuncts are true; the first row of the truth-table above is thus wrong. For example:

> Example (11): Publish or perish!
>
> Example (12): To be or not to be; that is the question.
>
> Example (13): Give me your money or I will kill you.

Cases like these exist and their existence shows the difference between logical language and ordinary language. 'Or' in natural

language is ambiguous. Sometimes it indicates what we call an inclusive 'or' relation, as in (10); sometimes it is used in the exclusive sense as in (11)–(13) and is called an exclusive 'or'. The inclusive 'or' is logically simpler and is more widely used in natural language. Hence, logic takes disjunctions by default to mean the inclusive 'or'. Indeed, the exclusive 'or' can be defined in terms of the inclusive 'or' – the exclusive 'or' is the inclusive 'or' plus the impossibility for both disjuncts to be true. Suppose we use the symbol $\oplus$ to represent the exclusive 'or', we can define this relation as:

$$p \oplus q =_{df} (p \vee q) \,\&\, \neg (p \,\&\, q)$$

$\neg (p \,\&\, q)$ means 'it is not the case that both p and q are true'. Brackets are used to indicate that the conjunction of p and q should take place before negation; thus the negation applies to the whole product of p conjoining q. This makes sure that the case of p and q being true is excluded from the relation of $(p \vee q)$. The truth-table of this exclusive disjunction relation is computed as follows:

p	q	$p \vee q$	$p \,\&\, q$	$\neg (p \,\&\, q)$	$(p \vee q) \,\&\, \neg (p \,\&\, q)$
T	T	T	T	F T	T F F
T	F	T	F	T F	T T T
F	T	T	F	T F	T T T
F	F	F	F	T F	F F T

Or put simply,

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

This is exactly the desired result. We will talk about computing truth-values of complex sentences from simple sentences very soon. For now, please do not worry if you do not understand how the truth-table above comes about.

The discussion above also demonstrates that logical symbols are more precise in meaning than ordinary language because ordinary language is often ambiguous. In case we cannot exactly match a logical relation with natural language, we translate the meaning of natural language into a set of logical symbols to reflect the required logical relation.

MATERIAL IMPLICATION ($\rightarrow$, $\supset$)

Material implication is commonly understood as a conditional indicated by the phrase 'if... then'. The component immediately following 'if' is called the *antecedent* whereas the other component or the component following 'then' is called the *consequent*. Note that in English, 'if' does not have to be at the beginning of a sentence and the word 'then' is sometimes omitted. The identification of the antecedent and the consequent is thus not subject to sentential structure. Even if the clause following 'if' is written at the end of the sentence, it is still an antecedent.

> Example (14): If it rains, then the ground is wet.
>
> (14') The ground is wet if it rains.
>
> Example (15): If the ground is wet, then it rains.

The above propositions are examples of material implication. (14) and (14') are equivalent, even though the word 'if' is situated in different positions. If we use p to stand for 'it rains' and q to stand for 'the ground is wet', both sentences represent the logical structure of '$p \rightarrow q$', read as 'p implies q'. (15), however, has a completely different logical structure. Its antecedent is 'the ground is wet' and its consequent is 'it rains'. Therefore, the logical structure is '$q \rightarrow p$', read as 'q implies p'. '$p \rightarrow q$' is different from '$q \rightarrow p$' in that they have different truth-tables (see the Spotlight box below), i.e. the complex propositions have different truth-values even when their atomic propositions have the same truth-values. Intuitively, (14) is true actually but (15) is not because it is possible that the ground is wet when there is no rain. For instance, someone may accidentally spill water on the ground or clean his car on that spot, so the ground is wet even though it does not rain.

We call a relation symmetric when '$p * q$' is equivalent to '$q * p$', where * represents any truth-functional connective. A relation that is not symmetric is asymmetric. Some truth-functional relations are symmetric; for example, 'p and q' is equivalent to 'q and p', 'p or q' is also equivalent to 'q or p'. However, material implication is not a symmetric relation. The one-sided arrow shows this feature of direction.

Material implication ($p \rightarrow q$) has the following truth-table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Material implication is characterized by the only case in which it is false, namely:

> A **material implication** is false only when the antecedent is true, but the consequent is false.

Spotlight: '$p \rightarrow q$' versus '$q \rightarrow p$'

$p \rightarrow q$ has the following truth-table:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

However, $q \rightarrow p$ has the following truth-table:

p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

They are different in two truth-value assignments, namely, the second and the third rows of each table. What does it mean? Take the second row as an example. When p is true and q is false, $p \rightarrow q$ is true while under the same condition, when p is true and q is false, $q \rightarrow p$ is false. In the $p \rightarrow q$ case, the row represents a situation in which something true (p) implies something false (q). According to the definition of material implication, it is false. However, in the $q \rightarrow p$ case, the row represents a situation in which something false (q) implies something true (p). According to the definition of material implication, any implication with a false antecedent is true. So the resultant truth-value is different. Similarly, the two tables are different in the third row. Two propositions are different if they have any difference in their truth-tables. Hence, these two propositions are different.

We can easily understand the first two rows of the truth-table of $p \rightarrow q$. When it rains and the ground is wet, it is obvious that the relation 'if it rains, then the ground is wet' holds. When it rains yet the ground is not wet, then it is equally obvious that the relation of 'if it rains, the ground is wet' does not hold. However, the last two rows of the truth-table are not so obvious. Logicians decide that when the antecedent is false, anything can happen and the implication is true regardless of the consequent. Thus, when p is false, no matter if q is true or false, the implication is true. This may seem puzzling at first, yet it actually makes sense. Let me explain.

Consider the following examples:

> Example (16): If 2 is greater than 3, then 2 squared is greater than 3 squared.
>
> Example (17): If you behave, then I will give you a candy.

(16) is a counterfactual conditional: it is in fact false that 2 is greater than 3 and it is also false that 2 squared (i.e. 4) is greater than 3 squared (i.e. 9). However, if 2 were greater than 3, it would be the case that the square of 2 is greater than the square of 3. It is because, in general, squaring a positive integer would form a greater integer. So, given one positive integer is greater than another, squaring it would only make the difference between these integers bigger. The relation in (16) is therefore true.

(17) is a promise. Suppose (17) is spoken by Tom's mummy to Tom. In the case that Tom behaves and Mummy gives him a candy, Mummy fulfils her promise and the conditional that 'if Tom behaves, then Mummy will give him a candy' is true. If Tom behaves but Mummy does not give him a candy, then Mummy breaks her promise and the conditional is false. However, when Tom does not behave and Mummy does not give him a candy either, Mummy does not break her promise. Her promise stays true, though the antecedent is not satisfied. Lastly, if Tom does not behave and Mummy gives him a candy, the promise is not broken either! Mummy may give Tom a candy for other reasons. It is never said that behaving is the *only* way to get a candy. In other words, the promise only governs what happens when the antecedent is fulfilled. When it is not fulfilled, Mummy is free to do anything without violating the promise. This captures well why logicians rule that in general a material implication is always true when the antecedent is false.

It may seem puzzling that an implication is true when the antecedent is false. It is because we tend to think of implication as something richer than a truth-functional relation. For example, we tend to regard an antecedent as an explanation to the consequent, or that implication indicates a causal relation. It hardly makes sense to explain something when what it is supposed to explain does not actually occur. Nor is it easy to imagine a causal relation when the cause is absent. A cause needs to precede its effect. Therefore, it is easy to understand the uneasiness we feel when the antecedent is false.

Key idea: Cause and explanation

Cause and explanation are related though different. Causal explanation is a kind of explanation, but explanation need not be causal. For instance, teleological explanation is often accepted as an explanation, yet it does not always provide a cause. Teleology is about the purpose or function, or what the agent aims to do. I may study for examinations because I want to graduate. The aim to graduate provides a purpose, or *telos*, to my action and the desire to graduate also provides a cause that drives it. So, *telos* provides the cause or motivation of action, mainly in autonomous agents

like humans. However, the heart may have the function to pump blood; yet this does not explain what causes the pumping. If we tell the heart: 'pump, or you don't fulfil your purpose', the heart will not start pumping! Instead, it is the whole mechanism of muscle contraction, relaxation, and coordination, etc. that explains the cause of pumping. Another example: wanting to eat the leaves on the top of trees does not cause a giraffe to grow a longer neck. Indeed, individual giraffes will not grow longer necks just because they want to. However, giraffes with longer necks do have an advantage in survival in that they may live longer and cause subsequent generations of giraffes to have longer necks. Evolution is thus explained by the causal mechanism rather than teleology.

To settle the uneasy feeling, we need to be aware of the point that material implication is indeed *not* an explanation or a causal relation. Material implication does not *have to* indicate any generalization or temporal order, though it may. A conditional is only *vacuously* true when the antecedent is false. It does not have to mean anything substantial or explain any actual happenings. Consider the variety of cases as follows.

> Example (18): If nothing is done to conserve energy, then all fossil fuels will be used up within a hundred years.

> Example (19): If there are holes in the walls, then there are termites.

> Example (20): Let me take you out for dinner if you pass your examination.

Raining is a cause for wet ground, but that nothing is done to conserve energy is not a cause for all fossil fuels being used up in a hundred years, at least not the *whole* cause for it. Other factors or conditions are necessary for the consequent to obtain, such as the limited supply of fossil fuels, the absence of an alternative energy source, the accelerating demand for energy consumption, etc. The antecedent is perhaps a factor to the consequent; however, the antecedent alone is insufficient. Hence, even if (18) is true, it does not follow that the antecedent causally *explains* the consequent.

In (19), holes in the walls are *not* the cause of termites at all. On the contrary, holes in the walls are the result of termite presence; they *indicate* the presence of termites. *The consequent is the cause* of the antecedent, rather than the other way around.

(20) has nothing to do with causation at all. It is an invitation and I can easily choose to honour or not honour my invitation for some *reason*. That I choose to do as I said, if at all, is a matter of free will. That does not mean I act randomly. I act for some reason. Yet reason is not the same as cause. This case shows that material implication may indicate *nothing about* causal relations.

In general, causal relation is a rich notion and is much more complicated than a truth-functional relation. A lot of conditions need to be established before a causal relation can be attributed. In common sense at least, a cause has to precede the effect. A cause also needs to be necessary in that if there is no such cause then no effect would come out.

In contrast, material implication is understood as only indicating that the presence of the antecedent is sufficient for the presence of the consequent. In logic, the technical term is that the antecedent is the **sufficient condition** for the consequent. 'p implies q' is true means that whenever p exists, q exists. Or, whenever p exists, it is not possible for q not to exist. Take (21) as an example. Suppose when you buy a ticket, you can enter a theatre and see the show. Then, buying a ticket is a sufficient condition for seeing the show.

Example (21): If you buy a ticket, you can see the show.

Conversely, when the absence of p is accompanied by the absence of q, i.e. whenever there is no p, there is no q, we say that p is the **necessary condition** of q. Using the example above again, buying a ticket is sufficient to get one in to see the show. However, it is not the necessary condition for seeing the show. Some people may not need to buy a ticket in order to see a show, such as invited guests of honour, or they may not even need a ticket at all, such as people wearing a staff permit. Yet being well is a necessary condition for seeing a show, for instance. If a person is sick, then he would not be able to enjoy

it. So (22) indicates the necessary relation between being well and seeing the show.

Example (22): If one is not well, then one cannot see the show.

(22') Only if one is well, then one can see the show.

We sometimes also indicate a necessary condition by using the words 'only if'. (22') asserts the same logical relation as (22).

Example (23): Being a member is necessary for enjoying club privileges.

(23') If one is not a member, then one cannot enjoy club privileges.

(23) and (23') are just another pair of examples of a necessary condition. Suppose a club is set up to serve members only. Being a member is the necessary condition for using the club. However, being a member is not the sufficient condition for enjoying club privileges because one may pay up a membership fee but never show up to enjoy any privileges.

That p is sufficient of q is the same as 'if p, then q', represented as:

$$p \rightarrow q$$

That p is necessary of q is the same as 'if not p, then not q', represented as:

$$\neg p \rightarrow \neg q$$

Or:

$$q \rightarrow p$$

We can prove that $\neg p \rightarrow \neg q$ is the same as $q \rightarrow p$ by a truth-table:

p	q	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The truth-tables of $q \rightarrow p$ and $\neg p \rightarrow \neg q$ are exactly the same.

We should also note that although p is sufficient for q, it does not imply that p is necessary for q. In fact, if p is sufficient for q, then q is necessary for p. For example, since raining is sufficient for wet ground, if the ground is not wet, then we can infer that there must be no rain. Therefore, wet ground is the necessary condition for raining. One may argue that it rains but the ground is dry because the ground is covered by some canopy, etc. But then in such a case, it would also render 'raining is sufficient for wet ground' untrue.

Conversely, if p is necessary for q, then q is sufficient for p. Let us use the membership example. Suppose being a member is necessary for enjoying club privileges, as stated in (23). Then, if a person enjoys club privileges, that person must be a member. This seems very straightforward. Note that although being a member is necessary for enjoying club privileges, it does not mean that being a member is sufficient for enjoying club privileges. A member may be too busy and never present at the facilities or services to enjoy the privileges.

So we have now explained an important pair of notions: necessary conditions and sufficient conditions, and their relationship with material implication. These concepts may be quite difficult to grasp, yet once mastered, they are actually very useful in identifying the correct logical relation represented by a natural sentence.

As a last remark, we need to say something more about conditionals. Material implication is a conditional proposition. However, not all conditional propositions are material implication. Indeed, there are many types of conditional in natural language. English has at least two types: the indicative conditional and the subjunctive conditional. Contrast the following:

> Example (24): *If I did* not smoke, I would not speak to you.
>
> Example (25): *If I hadn't* smoked, I *wouldn't have* spoken to you.

Suppose I only talk to people after I smoke, then the fact that I did not smoke would imply that I did not speak to anyone. However, it is perfectly possible that when I smoked, I would speak to someone. (24) is an indicative conditional, correctly

represented as material implication. (25), however, is not that simple. It is in fact a quote of Yul Brynner's, an actor who died of lung cancer because he smoked too much. He recorded this message so the footage could be used for an anti-smoking campaign after his death. It represents a counterfactual case: Brynner indeed smoked and died of smoking; yet given this fact, we imagine what could have happened otherwise. (25) is a subjunctive conditional because it grants the fact that the antecedent is false. Note more examples below.

> Example (26): If Oswald *did not shoot* Kennedy, then someone else *did*.

> Example (27): If Oswald *had not shot* Kennedy, then someone else *would have*.

Oswald indeed assassinated Kennedy. Suppose there might be more than one assassin assigned to kill him. (26) would be true in the situation in which if Oswald did not shoot, other assassins would. It leaves open the possibility that Oswald did not shoot and the truth of the conditional depends on the truth-values of the antecedent and the consequent. However, (27) admits the fact that Oswald did shoot. Suppose Oswald and another assassin both shot and Oswald's bullet reached Kennedy marginally earlier than another assassin's. So in the counterfactual case in which Oswald had not fired the shot, Kennedy would still die because the other assassin's bullet would reach him.

> Example (28): If it *is sunny*, Johnny will go out.

> Example (29): If it *were sunny*, Johnny would have gone out.

Whether a conditional is indicative or subjunctive (counterfactual) is typically indicated by the use of tense. Using perfect tense rather than the simple past (as in (24)–(27)), or past tense instead of the present tense (as in (28)–(29)), is a good indicator for the subjunctive sense.

The logical structure of subjunctive conditionals is more perplexing than material implication. For example, (30) and (31) are subjunctive conditionals, i.e. both have false antecedents. However, intuitively, (30) is false because China is not in Africa

so even if John were in China, he would not be in Africa. However, (31) seems true because China is indeed in Asia.

Example (30): If John were in China, he would be in Africa.

Example (31): If John were in China, he would be in Asia.

EQUIVALENCE (↔, ≡)

The final truth-functional connective to introduce in this book is equivalence. It has the following truth-table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Equivalence is characterized as follows:

Two propositions are **equivalent** if they *always* share the same truth-value.

Equivalence is sometimes denoted by the symbol '≡', suggesting strong equality. Equivalence is commonly translated as 'if and only if' in ordinary language and this indicates the biconditional relation. The double arrow symbol '↔' characterizes this feature. Consider an example:

Example (32): One can vote if and only if one is 18 years old.

What (32) means is that if a person can vote, they are 18 years old. Moreover, the only requirement for a person to vote is that the person is 18 years old. So being 18 is both the necessary and the sufficient conditions for voting. We can literally dissect the proposition as a conjunction of two conditional sentences:

(32') If one is 18 years old, one can vote.

(32") Only if one is 18 years old, can one vote.

(32') indicates that being 18 years old is sufficient for voting. This makes sure that people are not excluded from voting

because of other criteria, including social or economic factors. (32") indicates that being 18 years old is necessary for voting. This excludes all underage people from voting. So the two parts are different. (32") is the same as (32"') though. This is because if one must reach 18 in order to vote, then all those who can vote must be 18 years old or above. Being able to vote would then imply one is already 18 or more.

> (32"') If one can vote, then one is 18 years old.

Since equivalence indicates a biconditional relation, we can also write it as:

> $(p \equiv q)$ is equivalent to $[(p \rightarrow q) \ \& \ (q \rightarrow p)]$

Equivalence may not be commonly found in daily life; it is however philosophers' favourite when they try to define or analyse a concept. For example, a traditional understanding of knowledge is that knowledge is justified true belief. This idea is formulated formally as follows, where *S* stands for an agent and *p* a proposition:

> Example (33): *S* knows that *p* **if and only if** (i) *S* believes that *p*, (ii) *p* is true, and (iii) *S* is justified in believing *p*.

The formulation uses the connective of equivalence. Whether this analysis is true is a separate matter. Indeed, Edmund Gettier (1963) challenged its truth and sparked a lot of discussion.

PUNCTUATION – THE USE OF PARENTHESES

We mentioned at the beginning of Chapter 4 that a logic system, like a language, has three elements: syntax, semantics, and a set of rules governing how to use it, i.e. the rules of inference. Now we know the basic vocabularies of propositional logic, namely, atomic propositions and the five logical connectives defined by truth-tables. To make meaningful sentences (the technical term is well-formed formulas) in a logic system, we need some more rules in punctuation.

Parentheses are used in propositional logic, like punctuation, to show the order of the operations, i.e. which operation is to be performed first and what propositions are within the scope of

which operator. It is important to get the right order, because misplacing the parentheses would generate a totally different truth-value. It is similar to the case in arithmetic; for example, $(1 + 2) \times 3 = 9$ is different from $1 + (2 \times 3) = 7$. Similarly, $\neg (p \,\&\, q)$ is different from $\neg p \,\&\, q$. In the former case, p and q are to conjunct first before the whole result is negated, so the negation sign affects both p and q. However, in the latter case, negation only applies to p: p is to be negated first and then to conjunct with q.

In general, if a negation sign applies to an atomic proposition (represented by a small letter in italics), then no brackets are needed. Negation without brackets are always to be dealt with first. After that, ( ), [ ], and { } are signs to indicate operations in respective order; ( ) being the first to be dealt with, [ ] is the next, and so on.

We can now summarize the semantics and syntax of propositional logic as follows.

SEMANTICS

- Small letters (italicized in print) *a, b, c, d*... represent individual atomic propositions.
- Truth-functional connectives ¬ ∨, &, →, ↔ represent relations between atomic propositions. The meaning of each connective is defined by its truth-table.
- Brackets (), [], {} indicate the order of operations.

Connective	Ordinary language	Symbol	Logical feature
Negation	Not	¬	Opposite in truth-values
Conjunction	And	&	True only when both are true
Disjunction	Or	∨	False only when both are false
Material implication	If... then	→	False only when the antecedent is true and the consequent is false
Equivalence	... if and only if...	≡	True when both sides always have the same truth-value

The truth-table of each connective:

Negation

p	$\neg p$
T	F
F	T

Conjunction

p	q	$p \,\&\, q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Equivalence

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

SYNTAX

- Formulas are well-formed if and only if they have the following forms:

  a

  $\neg a$

  $a * b$, where * represents one of the truth-functional connectives. Technically, negation is a limiting case of $a * b$ where a is absent and b is connected by a negation function. So it can also be said that there are only two types of well-formed formula: a and $a * b$.

- Punctuation: Any operations without brackets should be done first, followed by (), [] and {} in respective order.

RULES OF INFERENCE

The validity of an argument is to be checked by constructing truth-tables. Truth-tables are exhaustive representations of all possible truth-value assignments of the propositions involved. Since it is exhaustive, it can be used as a proof for validity.

This chapter introduces two methods using truth-tables, one constructs a full truth-table for all propositions involved (Section 5.5) while the other one uses a shortened truth-table to check validity (Section 5.6).

Like categorical logic, there is also a rule-theoretic model of propositional logic, which deduces valid inferences by applying rules step by step (Section 5.8). This method is called natural deduction. While some may prefer this method because it resembles the actual thought processes in some aspects, the full truth-table method is exhaustive, mechanical and certain in getting results. We will try to master the truth-table method first.

5.3 Computing truth-tables

Having introduced all the basic elements of propositional logic above, we are now in a position to explain how to compute truth-tables of complex propositions systematically from simpler propositions. I have already done some computation when I explain the exclusive 'or' and equivalence. The principles of computation used are the same.

STEPS TO CONSTRUCT A TRUTH-TABLE FROM ATOMIC PROPOSITIONS

1 Decide how many rows this truth-table should have (2^n where n = number of atomic proposition involved).

2 Write down all possibilities of combination at the left hand side of a table.

3 Compute the values of each operation, starting from the most basic ones and work out according to the order specified by parentheses, if any.

The number of rows needed for a complex proposition depends on the number of atomic propositions involved. Each atomic proposition has two possible truth-values: true or false. Hence the possible truth-value combination of a complex proposition is 2^n, n being the number of atomic propositions.

For illustration, suppose we want to compute the truth-table of:

Example (34): $p \ \& \neg q$

It involves two atomic propositions, p and q. Therefore, its truth-table has 2^2, i.e. 4, rows to exhaust all possible truth-value assignments. So we draw a table of four rows and write down each atomic proposition on the left-hand side, leaving the complex proposition on the right.

p	q	$p \,\&\, \neg q$

Now we list out all possible truth-value combinations of the atomic propositions on the left. We should do this systematically so that we don't miss out any combination. Since each proposition has only two values, let us adopt the habit of splitting the rows half and half for each atomic proposition. Namely, if there are two atomic propositions, we write under the first atomic proposition T in the first two rows, then F for the rest. Then we write under the second atomic proposition T and F alternately. If there are three atomic propositions, we write four Ts and four Fs for the first proposition, then two Ts and two Fs under the second proposition, then T and F under the third one. So on and so forth. Our table becomes:

p	q	$p \,\&\, \neg q$
T	T	
T	F	
F	T	
F	F	

The two columns on the left specify the conditions of truth-value assignment for each row. They are not the truth-values of the complex proposition. We can read across a row of a truth-table in this way: if p is T and q is F, then $p \,\&\, \neg q$ is...; whereas if p is T and q is F, then $p \,\&\, \neg q$ is..., etc.

$p \,\&\, \neg q$ involves the operations of negation and conjunction. But which operation goes first? From our discussion of punctuation above, we know that negation applied to an atomic proposition

always goes first. So to compute the truth-table of $p \mathbin{\&} \neg q$, we first compute $\neg q$ and then conjunct it with p. We can do this either by adding a column $\neg q$ or working directly under the column of $p \mathbin{\&} \neg q$. If the former way is adopted, be aware that this is an intermediate step we add for convenience and not to confuse it with a given premise in an argument. When we use truth-tables to evaluate arguments, each column represents a premise or a conclusion given in an argument. So it is neater to work directly under the column of $p \mathbin{\&} \neg q$.

We follow a convention of writing the truth-value of each proposition after an operation immediately under the *operation sign* (not the proposition sign). It is important not to muddle up the positions. We should also copy the values of the atomic propositions from the left columns before computing.

Copy the values of q and then negate them. We get the following truth-table:

p	q	$p \mathbin{\&} \neg q$
T	T	F T
T	F	T F
F	T	F T
F	F	T F

We then conjunct p with $\neg q$. We copy the truth-values of p from the left-hand columns.

p	q	$p \mathbin{\&} \neg q$
T	T	T F T
T	F	T T F
F	T	F F T
F	F	F T F

Now, we compute the truth-value of each conjunction. In the first row, we conjunct something true (p) with something false ($\neg q$); the result is false because a conjunction is true only if both sides are true. The second row has two true conjuncts, so the result is true. Similarly, the third row is false and the fourth row is false, too. We write these results directly under the conjunction sign because they are the results after this operation. This is the

truth-table for the complex proposition p & ¬ q. I have highlighted the result by underlining the final truth-values.

p	q	p & ¬ q
T	T	T $\underline{F}$ F T
T	F	T $\underline{T}$ T F
F	T	F $\underline{F}$ F T
F	F	F $\underline{F}$ T F

The above truth-table is to be interpreted as:

> If p is true and q is true, then p & ¬ q is false.
>
> If p is true and q is false, then p & ¬ q is true.
>
> If p is false and q is true, then p & ¬ q is false.
>
> If p is false and q is false, then p & ¬ q is false.

The above does not involve the use of brackets. Or, we may say that brackets have already implicitly been applied, i.e. (p & ¬ q). Now let us try some examples that use brackets. They also serve to illustrate how the use of parentheses affects the computation of truth-values, and thus the meaning or the identification, of a proposition.

> Example (35): ¬ (p & q)

Like (34), (35) also involves the operations of negation and conjunction. However, now the bracket covers both p and q. This means that the conjunction of p and q has to be done first before the whole thing is negated. As we shall see, this change of order completely alters the truth-values of the complex propositions. (34) and (35) are thus completely different propositions.

(35) involves two atomic propositions p and q; hence its truth-table has four rows.

p	q	¬ (p & q)
T	T	
T	F	
F	T	
F	F	

We compute the conjunction of p and q first. When p is true and q is true, the conjunction should be true. In all other cases, the conjunction is false because at least one atomic proposition is false. Put the truth-values immediately under the conjunction sign, so we have the following table:

p	q	$\neg (p \mathbin{\&} q)$
T	T	T T T
T	F	T F F
F	T	F F T
F	F	F F F

We then compute the negation. Negation gives opposite truth-values, so the following table is obtained. The complex proposition involves only two operations, so we have completed the task.

p	q	$\neg (p \mathbin{\&} q)$
T	T	F T T T
T	F	T T F F
F	T	T F F T
F	F	T F F F

Compare this result with that of (34). It is now clear how they are different propositions because they have different truth-values under the same truth-values of respective atomic propositions.

Example (36): $\neg p \mathbin{\&} \neg q$

One may wonder whether (35) and (36) are the same. They are not. Let us show this by computing the truth-table of (36) and comparing it to that of (35).

p	q	$\neg p \mathbin{\&} \neg q$
T	T	
T	F	
F	T	
F	F	

Negation applies to the immediately preceding proposition, so we compute $\neg p$ and $\neg q$ respectively first.

p	q	$\neg p \& \neg q$
T	T	FT FT
T	F	FT TF
F	T	TF FT
F	F	TF TF

Then compute the conjunction of two negated propositions. This is also the last operation of this complex proposition.

p	q	$\neg p \& \neg q$
T	T	FT$\underline{\text{F}}$FT
T	F	FT$\underline{\text{F}}$TF
F	T	TF$\underline{\text{F}}$FT
F	F	TF$\underline{\text{T}}$TF

Compare the result with that of (35). They are different and so the two propositions are different. Indeed, (34), (35) and (36) are all different. However, sometimes propositions looking different are indeed equivalent. Consider (37).

Example (37): $\neg p \vee \neg q$

The punctuation requires that we compute $\neg p$ and $\neg q$ respectively, then find the disjunction of them.

p	q	$\neg p \vee \neg q$
T	T	FT FT
T	F	FT TF
F	T	TF FT
F	F	TF TF

A disjunction is false only when both disjuncts are false. The truth-table of (37) is thus as follows:

p	q	$\neg p \vee \neg q$
T	T	FT$\underline{\text{F}}$FT
T	F	FT$\underline{\text{T}}$TF
F	T	TF$\underline{\text{T}}$FT
F	F	TF$\underline{\text{T}}$TF

Compare the truth-tables of (35) and (37). They are exactly the same! So (35) and (37) are **truth-functionally equivalent**. Logic concerns only truth-values. Hence in logic, two propositions are equivalent if and only if all of their possible truth-value assignments are exactly the same.

All the steps above are drawn for illustrative purposes only. The goal is to draw one last truth-table. Let us now do an exercise for practice.

Exercise 5.1: Drawing truth-tables

Draw the truth-tables of the following propositions.

1. $p \rightarrow (\neg p \lor p)$
2. $(p \,\&\, \neg p) \equiv p$
3. $p \rightarrow \neg q$
4. $\neg p \lor q$
5. $\neg (p \rightarrow q)$
6. $p \equiv [q \lor (\neg q \rightarrow p)]$
7. $p \,\&\, \neg (q \rightarrow r)$
8. $(p \,\&\, \neg q) \rightarrow r$
9. $p \rightarrow (q \rightarrow r)$
10. $(p \rightarrow q) \rightarrow r$
11. $(p \rightarrow q) \lor (r \rightarrow \neg q)$
12. $(p \,\&\, q) \rightarrow (\neg q \rightarrow \neg r)$
13. $(p \equiv r) \rightarrow [(p \rightarrow q) \,\&\, (\neg r \rightarrow \neg q)]$

Questions 1–5 are pretty straightforward. You may just check the answer in the back. Questions 6–8 are slightly more complex, so let us go through them together below. Once you master them, you can surely handle all the rest and please just check the solutions for Questions 9–13.

For the examples above, we have only gone through propositions with one set of parentheses. Yet Question 6 has two. How shall we deal with it? Yes, as suggested before, we deal with any parentheses in respective order: first (), then [], then {}.

Question 6: $p \equiv [q \lor (\neg q \rightarrow p)]$

There are only two atomic propositions. So, there are four rows.

p	q	$p \equiv [q \vee (\neg q \to p)]$
T	T	
T	F	
F	T	
F	F	

The correct order: do the negation within the small bracket first, then compute the implication within it; after finishing all operations within the small bracket, settle the middle bracket. Finally, solve the equivalence operation outside the brackets, which is the last operation in this proposition.

p	q	$p \equiv [q \vee (\neg q \to p)]$
T	T	F T
T	F	T F
F	T	F T
F	F	T F

Compute the implication between $\neg q$ and p.

p	q	$p \equiv [q \vee (\neg q \to p)]$
T	T	F T T T
T	F	T F T T
F	T	F T T F
F	F	T F F F

Now settle the middle bracket: use the values under the implication sign to disjunct with q. Put the values under the disjunction sign.

p	q	$p \equiv [q \vee (\neg q \to p)]$
T	T	T T F T T T
T	F	F T T F T T
F	T	T T F T T F
F	F	F F T F F F

Finally, use the values under the disjunction sign to compute the equivalence relation with p. The underlined letters mark the final truth-table of the complex proposition in question.

p	q	$p \equiv [q \vee (\neg q \rightarrow p)]$
T	T	T $\underline{T}$ TT FTTT
T	F	T $\underline{T}$ FT TFTT
F	T	F $\underline{F}$ TT FTTF
F	F	F $\underline{T}$ FF TFFF

Question 7 and Question 8 involve not two but three atomic propositions. When three atomic propositions are involved, we need 2^3, i.e. 8, rows. The truth-value assignments are systematically split in the half and half manner as shown in the following table.

Question 7: $p \mathbin{\&} \neg (q \rightarrow r)$

p	q	r	$p \mathbin{\&} \neg (q \rightarrow r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Other procedures should be standard. We deal with the small brackets first. So, implication, then negation, then conjunction.

The implication:

p	q	r	$p \mathbin{\&} \neg (q \rightarrow r)$
T	T	T	TTT
T	T	F	TFF
T	F	T	FTT
T	F	F	FTF
F	T	T	TTT
F	T	F	TFF
F	F	T	FTT
F	F	F	FTF

The negation:

p	q	r	$p \mathbin{\&} \neg (q \rightarrow r)$
T	T	T	F T T T
T	T	F	T T F F
T	F	T	F F T T
T	F	F	F F T F
F	T	T	F T T T
F	T	F	T T F F
F	F	T	F F T T
F	F	F	F F T F

Now the conjunction:

p	q	r	$p \mathbin{\&} \neg (q \rightarrow r)$
T	T	T	T F F T T T
T	T	F	T T T T F F
T	F	T	T F F F T T
T	F	F	T F F F T F
F	T	T	F F F T T T
F	T	F	F F T T F F
F	F	T	F F F F T T
F	F	F	F F F F T F

Question 8 is quite similar to Question 7 except that the brackets are in different places. This affects the order of computation. We shall now do: negation first, then conjunction, then implication. Still eight rows are needed because there are three atomic propositions.

Question 8: $(p \mathbin{\&} \neg q) \rightarrow r$

p	q	r	$(p \mathbin{\&} \neg q) \rightarrow r$
T	T	T	T F F T
T	T	F	T F F T
T	F	T	T T T F
T	F	F	T T T F
F	T	T	F F F T

p	q	r	$(p \,\&\, \neg q) \rightarrow r$
F	T	F	F F F T
F	F	T	F F T F
F	F	F	F F T F

p	q	r	$(p \,\&\, \neg q) \rightarrow r$
T	T	T	T F F T $\underline{T}$ T
T	T	F	T F F T $\underline{T}$ F
T	F	T	T T T F $\underline{T}$ T
T	F	F	T T T F $\underline{F}$ F
F	T	T	F F F T $\underline{T}$ T
F	T	F	F F F T $\underline{T}$ F
F	F	T	F F T F $\underline{T}$ T
F	F	F	F F T F $\underline{T}$ F

Note again that the truth-tables of Question 7 and Question 8 are different, so the propositions they contain are different propositions.

Spotlight: Five truth-functional connectives

We have now explored five truth-functional connectives: negation, conjunction, disjunction, material implication and material equivalence. We have also discussed whether we absolutely need five connectives in propositional logic. The answer is no, because some relations can be expressed in terms of other relations. The rule is that two relations are equivalent if and only if their truth-tables are identical. Now we have learned about truth-tables, I can show you how these reductions are done, as follows.

Equivalence is reducible to implication and conjunction:

p	q	$p \leftrightarrow q$	$(p \rightarrow q) \,\&\, (q \rightarrow p)$
T	T	T $\underline{T}$ T	T T T $\underline{T}$ T T T
T	F	T $\underline{F}$ F	T F F $\underline{F}$ F T T
F	T	F $\underline{F}$ T	F T T $\underline{F}$ T F F
F	F	F $\underline{T}$ F	F T F $\underline{T}$ F T F

Material implication is reducible to negation and disjunction:

p	q	$p \rightarrow q$	$\neg p \vee q$
T	T	T T T	F T T T
T	F	T F F	F T F F
F	T	F T T	T F T T
F	F	F T F	T F T F

The Sheffer stroke is defined as the negation of conjunction and is represented by the following truth-table (the fourth column). It literally means at least one proposition is false. It is equivalent to the disjunction of negations (the fifth column).

p	q	$\neg (p \& q)$	$p \mid q$	$\neg p \vee \neg q$
T	T	F T T T	T F T	F T F F T
T	F	T T F F	T T F	F T T T F
F	T	T F F T	F T T	T F T F T
F	F	T F F F	F T F	T F T T F

Other connectives can be defined in the Sheffer stroke. For instance, $\neg p$ is equivalent to $p \mid p$:

p	$\neg p$	$p \mid p$
T	F T	T F T
F	T F	F T F

$p \vee q$ is equivalent to $(p \mid p) \mid (q \mid q)$.

$p \& q$ is equivalent to $(p \mid q) \mid (p \mid q)$.

$p \rightarrow q$ is equivalent to $p \mid (q \mid q)$ or $p \mid (p \mid q)$.

Equivalence is reducible to implication and conjunction, both of which are reducible to the Sheffer stroke. Hence, it is obvious that equivalence is also reducible to the Sheffer stroke.

p	q	$(p \mid p) \mid (q \mid q)$	$(p \mid q) \mid (p \mid q)$	$p \mid (q \mid q)$	$p \mid (p \mid q)$
T	T	T F T T T F T	T F T T T F T	T T T F T	T T T F T
T	F	T F T T F T F	T T F F T T F	T F F T F	T F T T F
F	T	F T F T T F T	F T T F F T T	F T T F T	F T F T T
F	F	F T F F F T F	F T F F F T F	F T F T F	F T F T F

Peirce's arrow is defined as the negation of disjunction. This means that neither p nor q is true.

p	q	$\neg (p \vee q)$	$p \perp q$
T	T	F T T T	T F T
T	F	F T T F	T F F
F	T	F F T T	F F T
F	F	T F F F	F T F

You may likewise prove that:

$\neg p$ is equivalent to $p \perp p$.

$p \vee q$ is equivalent to $(p \perp q) \perp (p \perp q)$.

$p \,\&\, q$ is equivalent to $(p \perp p) \perp (q \perp q)$.

$p \rightarrow q$ is equivalent to $((p \perp p) \perp q) \perp ((p \perp p) \perp q)$.

5.4 Translating ordinary language

Logic is related to natural language and reasoning. Let us go through some rules for translating our daily arguments into propositional logic.

▶ Use of brackets

We have already explained the importance of parentheses in propositional logic. However, we do not use brackets in ordinary language. Therefore, we can only judge their presence through interpreting texts in context.

Example (38): If there are no protests, then there is no military intervention and the government is not challenged.

Example (39): If there are no protests and politicians do not speak up for the people, then the government is not challenged and there is no military intervention.

Example (40): The government is challenged provided that either politicians speak for the people or there are protests.

Example (41): There is military intervention or the government is challenged if there are protests.

Suppose p = there are protests, q = there is military intervention, r = the government is challenged, s = politicians speak up for the people. The above should be symbolized respectively, as follows.

(38') $\neg p \rightarrow (\neg q \ \& \ \neg r)$

(39') $(\neg p \ \& \ \neg s) \rightarrow (\neg r \ \& \ \neg q)$

(40') $(s \vee p) \rightarrow r$

(41') $q \vee (p \rightarrow r)$

(41") $p \rightarrow (q \vee r)$

In (38), 'there is no military intervention' and 'the government is not challenged' are both consequences of the absence of protests. Hence, a bracket should apply to conjunct the two propositions before taking the whole conjunction as the consequent of the conditional. The punctuation (location of commas, for instance) helps to indicate such meaning.

In (39), the word 'and' is found within the antecedent of the main complex proposition. So the antecedent includes the conjunction of the two atomic propositions. A set of brackets should apply to the antecedent. The consequent of this conditional also contains two atomic propositions, so another set of brackets is necessary.

The main operation of (40) is a conditional, indicated by the phrase 'provided that'. Now the antecedent is in the rear part of the sentence but it contains a disjunction. A set of brackets should apply there.

(41) is translated either as $q \vee (p \rightarrow r)$, in which the main operator is 'or', or it can also be translated as $p \rightarrow (q \vee r)$, where the main operator is 'if'. Interestingly, the two formulations are indeed equivalent. So both are correct readings.

Sometimes, if a sentence is really ambiguous and it is impossible to clarify with the speaker, we would have to appeal to linguistic understanding and exercise our judgement to decide what is the best or most natural reading.

▶ 'But'

'But' works the same in logic as 'and'. So do other conjunction words such as 'however', 'nevertheless', 'despite', 'although', etc. These words play different rhetorical roles to indicate contrasts, disagreement, etc. and in doing so, often affect the audience's psychology and expectation. Yet as far as truth is concerned, they still demand that all constituents are true. So they play the same logical role as 'and'.

> Example (42): Peter is a cheater but John is not.
>
> Let p = Peter is a cheater, j = John is a cheater. (42) is symbolized as:
>
> (42') $p \ \& \neg j$

▶ 'Either... or...'

'Either… or…' is understood logically as the inclusive 'or', unless it clearly means otherwise in context.

> Example (43): Either a parent or a child's legal guardian can enjoy tax credits on childcare.
>
> Let p = a parent can enjoy tax credits on childcare, q = a child's legal guardian can enjoy tax credits on childcare. (43) is symbolized as:
>
> (43') $p \vee q$

▶ 'Only if'

A sentence of the form 'if p, then q' is standardly translated as a case of material implication '$p \rightarrow q$', in which p is the sufficient condition for q. 'Only if' is also material implication; however, the clause immediately following 'only if' is a necessary rather than a sufficient condition for the other clause. Hence, in general, a sentence of the form 'only if p, then q' should be translated as '$\neg p \rightarrow \neg q$' or simply, '$q \rightarrow p$'. For instance:

> Example (44): Only if John loves Mary wholeheartedly would Mary agree to marry him.

Suppose (44) is true. John's loving Mary wholeheartedly is a necessary condition for Mary's agreeing to marry John. So if Mary agrees to marry John, it must be the case that John loves Mary wholeheartedly.

Let p = John loves Mary wholeheartedly, q = Mary agrees to marry John. (44) is symbolized as follows:

(44') $q \rightarrow p$

Alternatively, if John does not love Mary wholeheartedly, then Mary would not agree to marry him. So (44) can also be symbolized, equivalently, as:

(44") $\neg p \rightarrow \neg q$

▶ 'Unless'

Example (45): Unless you give me a diamond ring, (otherwise) I will not marry you.

'Unless' is slightly tricky. Like 'only if', it indicates a necessary condition. However, the part followed by 'unless' is usually negative. For example, (45) indicates that giving a diamond ring is a necessary condition for a wedding proposal. So, if (45) is true, the following situations must be true: (i) If you don't give me a diamond ring, then I will not marry you; (ii) If I marry you, you must have already given me a diamond ring; (iii) Either you give me a diamond ring, or I will not marry you. All these three cases are equivalent to (45). So we can translate a sentence including 'unless' in the following ways.

Let p = you give me a diamond ring, q = I will marry you.

Unless p, not q.

(45') $\neg p \rightarrow \neg q$

(45") $q \rightarrow p$

(45"') $p \vee \neg q$

However, it is only a linguistic convention that 'unless' is usually followed by a negative clause. If we consider the general

logical form related to 'unless', then we should state the rule of translation about 'unless' as follows:

'Unless p, q' is translated as either: $\neg p \rightarrow q$, $\neg q \rightarrow p$, $p \vee q$

To avoid confusion, personally I prefer to use the last formula. That is, the operation related to 'unless' is simply a disjunction: 'Unless p, q' is '$p \vee q$'.

To test our understanding, let us try translating the following. Do these propositions express the same logical form?

Example (46): Either Peter is sick, or he will go to the party.

Example (47): Unless Peter is sick, he will go to the party.

Example (48): Only if Peter is sick, then he will not go to the party.

Example (49): If Peter is sick, then he will not go to the party

Let p = Peter is sick, q = Peter will go to the party. (46) can be symbolized very straightforwardly as:

(46') $p \vee q$

(47) also represents the situation that either Peter is sick or he will go to the party. Being sick is the only reason (the necessary condition) for his absence from the party. So, if Peter is not sick, then he surely will go to the party. It also implies: if Peter is not at the party, he must be sick. The following are all valid translations of (47) and they are indeed equivalent in truth-values.

(47') $p \vee q$

(47'') $\neg p \rightarrow q$

(47''') $\neg q \rightarrow p$

In (48), 'only if' signifies that Peter's being sick is the only situation in which he will not go to the party. So being sick is the necessary condition of his absence from the party. (48) shares the same logical form as (47) and can be symbolized as:

(48') $\neg q \rightarrow p$

So, (46)–(48) are in fact the same proposition expressed in different ways. Is (49) also the same proposition? No! (49) expresses that Peter's being sick is *sufficient* for his absence to the party. It does not indicate that being sick is a necessary condition. Peter may not go to the party for other reasons; being sick is just one of them. So (49) is a different proposition and it can be symbolized as:

(49') $p \rightarrow \neg q$

The translation of propositional logic is probably more straightforward than that of categorical logic. The only point of caution is to look out for sufficient and necessary conditions. 'Only if' and 'unless' indicate necessary conditions. Now, let us summarize the rules of translation learned so far and then try an exercise.

Key idea: Translation in propositional logic

Rules

- **But:** the same as 'and'
- **Either... or**: the same as 'or'
- **If:** 'If A, then B': $A \rightarrow B$ (also equivalent to $\neg A \vee B$)
- **Only if**: 'Only if A, then B': $B \rightarrow A$ (also equivalent to $\neg A \rightarrow \neg B$, or $A \vee \neg B$)
- **Unless**: 'Unless A, B': $A \vee B$ (also equivalent to $\neg B \rightarrow A$, or $\neg A \rightarrow B$)
- 'Unless A, not B': $A \vee \neg B$ (also equivalent to $B \rightarrow A$, or $\neg A \rightarrow \neg B$)
- **Only if A, then B:** the same as 'Unless A, not B'

Necessary and sufficient conditions

- **If A then B**: A is the sufficient condition for B.
- **Only if A, then B**: A is the necessary condition for B. If A had *not* been the case, then B would *not* be the case either.
- **If A is sufficient for B, then B is necessary for A.**

Exercise 5.2: Translating truth-functional connectives

Translate the following sentences into symbols and draw their truth-tables.

1 Either Saudi Arabia buys 500 warplanes and Iran raises the oil price or Jordan requests more American aid.
2 It is not the case that either Egypt's food shortage worsens or Jordan requests more American aid.
3 That he has a good lawyer implies that he will be acquitted.
4 If you deceive or coerce another human being, you violate her rights.
5 Britain joined the war in Iraq if and only if the USA declared it.
6 Only if I see his wound and touch his scar, then I would believe in him.
7 Provided that neither Chile nor the Dominican Republic calls for a meeting of the Latin American states, Brazil will protest to the UN if Argentina mobilizes.
8 Unless you have superpowers, you cannot get away from the situation and be unharmed.

5.5 Testing validity

We are now familiar with translating ordinary language into symbols and operating on the symbols to find out the truth-tables of each complex proposition. With these skills at hand, we are ready to learn how to test the validity of arguments.

Rehearse the definition of validity of an argument. *A valid argument guarantees the truth of the conclusion when all premises are true.* If we check that in all cases in which the premises are true, the conclusion is also true, then the definition is satisfied, that is, the argument is valid. Anything that is not valid is invalid. We are going to do that checking.

A truth-table reveals all possible truth-value assignments of a proposition. An argument contains several propositions. We can

then simply put them side by side and construct a large truth-table. Then we can check all truth-value assignments to see whether in all cases where all premises are true, the conclusion is also true. The number of rows of a truth-table depends on the total number of atomic propositions that appear in the argument. The 2^n rule applies.

Propositional logic can deal with any number of premises and so it is not restricted to syllogism. This makes propositional logic more available than categorical logic. Propositional logic requires that the more atomic propositions are involved, the more rows are needed in the truth-table (the number of rows = 2^n). Hence, if a lot of atomic propositions are involved in an argument, then a truth-table may become so long that technically, it is difficult for humans to draw and check it. However, with advances in technology, the difficulty can be easily overcome by leaving a computer program to do all the calculation and checking. We describe propositional logic as **computable**, i.e. there is an effective procedure for calculating a solution. This is indeed an advantage of propositional logic.

Let us list the steps for validity checking in propositional logic as follows.

▶ Steps for validity checking in propositional logic

1 Translate the argument into symbols.

- ▷ Identify the conclusion and the premises.
- ▷ Identify all atomic propositions and symbolize them.
- ▷ Identify the connectives among them.
- ▷ Symbolize all premises and the conclusion.

2 Draw all premises and conclusion into a truth-table.

- ▷ Decide how many rows are needed in total.
- ▷ List all possible truth-value assignments.
- ▷ Calculate the truth-values of each proposition under each possible assignment.

3 Decide validity, i.e. check whether whenever the premises are true, the conclusion is true; or equivalently, whether there is an assignment making the conclusion false but all premises true.

SOME COMMON VALID ARGUMENT FORMS AND FALLACIES

Let us try out some examples. As usual, we will look at the simplest first and move on to more complex ones.

> Example (50): If it is sunny, then we go to the beach. It is sunny. Therefore, we go to the beach.

First, identify the conclusion and the premises. The conclusion is 'we go to the beach'. The premises are the remaining two sentences. Then we identify the atomic propositions involved. There are two atomic propositions in this argument: 'it is sunny' and 'we go to the beach'. We can symbolize the argument as follows.

Let p = it is sunny and q = we go to the beach.

$p \to q$

$\underline{p\quad\quad}$

$\therefore q$

Since there are two atomic propositions, the number of rows needed for drawing the truth-table of this argument is $2^2 = 4$. This truth-table should contain two columns for the respective atomic propositions. Then each proposition (premises and the conclusion) should occupy a column. We thus set a truth-table as follows.

p	q	$p \to q$	p	q
T	T			
T	F			
F	T			
F	F			

We now compute the truth-values of each proposition under each possible assignment. For $p \to q$, when p is T, q is T, $p \to q$ should be T. When p is T, q is F, $p \to q$ should be F. So on and so forth. Remember to put the truth-value exactly under the

connective to indicate that this is the truth-value assignment after the relevant operation is performed. The last two columns should be easy; we just copy the truth-values from the left-hand side.

p	q	$p \to q$	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Now the truth-table is completed. We just need to determine validity. An argument is valid if and only if the premises are true, and the conclusion is true. The premises here are $p \to q$ and p, so we go through the table to pick out all cases in which they are both true. Then we check whether in all these cases the conclusion, namely q, is true. There is only one such case in which the premises are all true and it is the first row. In that case, the conclusion is also true. We thus conclude that for this argument, whenever all premises are true, the conclusion is true. The argument is valid.

The claim 'In every case in which all premises are true, the conclusion is true' is equivalent to 'there is no case in which all premises are true but the conclusion is false'. Hence, instead of looking for all cases in which all premises are true, we can also determine validity simply by looking for a single case in which all premises are true but the conclusion is false. If there is such a case, then the argument is invalid. If there is not any such case, then the argument is valid. This way of looking is handy when there are a lot of rows to check, though it may not make much difference in this simple example. In (50), there is no case in which the premises are all true and the conclusion is false. Hence, the argument is not invalid, i.e. it is valid.

(50) is a very simple but incredibly common argument form. It is so useful and common that logicians have given this valid argument form a name, in Latin: ***modus ponens.***

> Example (51): If it is sunny, then we go to the beach. We do not go to the beach. Therefore, it is not sunny.

(51) represents another extremely common argument form named ***modus tollens***. Let us perform the same procedures. First, identify the conclusion and the premises. 'It is not sunny' is the conclusion; the other two sentences are the premises. The argument is symbolized as follows: p and q stand for the same atomic proposition as stated in (50).

$$p \to q$$

$$\underline{\neg q}$$

$$\therefore \neg p$$

Second, draw the truth-table. There are two atomic propositions; so we need four rows. The truth-table is constructed as follows.

p	q	$p \to q$	$\neg q$	$\neg p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

There is only one case in which the premises are all true, namely the fourth row. In that case, the conclusion is also true. Therefore, we conclude that for this argument, whenever all premises are true, the conclusion must be true. This argument is valid.

Modus ponens and *modus tollens* are valid argument forms. However, there are common fallacies that look like them but are not and those arguments are invalid. (52) and (53) are examples of them.

> Example (52): If it is sunny, then we go to the beach. It is not sunny. Therefore, we do not go to the beach.

Identify the conclusion and the premises. The same atomic propositions are involved, so we adopt the same set of symbols as before. The argument is symbolized as follows.

$$p \to q$$

$$\underline{\neg p}$$

$$\therefore \neg q$$

The truth-table:

p	q	$p \to q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	<u>T</u>	<u>T</u>	<u>F</u>
F	F	T	T	T

There are two cases in which the premises are all true, namely the third row and the fourth row. The conclusion is true in the fourth row. However, it is false in the third. We highlight it by underlining the truth-values. Since it is possible that the premises are all true but the conclusion is false (the third row), the argument does not fulfil the requirement of validity which excludes such a possibility. The argument is thus invalid.

You may wonder why only one false case can render the argument invalid. There is a case in which the conclusion is true when the premises are all true. Does it not make the argument valid? The answer is no. Validity requires certainty. We may not know whether the premises are indeed true or not, but we need to ensure that in all possible cases where the premises are true, the conclusion must follow. So even one single case of falsehood is sufficient to judge that an argument is invalid. It does not matter whether there are other cases in which the premises are true and the conclusion is true. As long as there is no guarantee for the truth of the conclusion when premises are true, an argument is invalid.

The argument form in (52) may look like that of (51), but it is not. (51) denies the consequent, whereas (52) denies the antecedent. An antecedent is the sufficient condition for the consequent; it is not the necessary condition. There could be other conditions also sufficient to lead to the consequent. Hence, denying the antecedent would not necessarily lead to denial of the consequent. Arguments of the form like (52) commit a fallacy called **the fallacy of denying the antecedent.**

> Example (53): If it is sunny, then we go to the beach. We go to the beach. Therefore, it is sunny.

Using the same set of symbols, (53) is symbolized and its truth-table constructed as follows.

$$\begin{array}{l} p \to q \\ \underline{q\quad\ } \\ \therefore p \end{array}$$

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	<u>T</u>	<u>T</u>	<u>F</u>
F	F	T	F	F

The argument is invalid because it is possible for all premises to be true yet the conclusion is false, as in the third row. It is true that in the first row when all premises are true, the conclusion is true. However, for reasons explained above, one falsehood is sufficient to deny the validity of an argument. This argument remains invalid.

The fallacy committed by this argument form is called the **fallacy of affirming the consequent.** It is valid to affirm the antecedent but it is fallacious to affirm the consequent. It is because many conditions may lead to the consequent, merely affirming the consequent does not guarantee the antecedent is true.

In the following examples, other common argument forms and fallacies are to be introduced and proved by truth-tables.

> Example (54): If governments do not regulate carbon emissions, then global warming will increase. If global warming increases, then super-catastrophes will emerge. Therefore, if governments do not regulate carbon emissions, then super-catastrophes will emerge.

The last sentence is the conclusion and the first two sentences are the premises. There are altogether three atomic propositions. So there should be $2^3 = 8$ rows in the truth-table.

Let p = governments regulate carbon emissions, q = global warming increases, r = super-catastrophes emerge.

$$
\begin{array}{l}
p \to q \\
\underline{q \to r} \\
\therefore p \to r
\end{array}
$$

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T T T	T T T	T T T
T	T	F	T T T	T F F	T F F
T	F	T	T F F	F T T	T T T
T	F	F	T F F	F T F	T F F
F	T	T	F T T	T T T	F T T
F	T	F	F T T	T F F	F T F
F	F	T	F T F	F T T	F T T
F	F	F	F T F	F T F	F T F

To determine the validity, we look at the truth-values of each complex proposition. There are several cases in which all premises are true, namely, the first, the fifth, the seventh, and the eighth row. In all these cases, the conclusion is true. Hence, it is not possible for the conclusion to be false when all premises are true. The argument is valid.

This argument form is quite intuitively valid. It is like a chain which links p to q and q to r. q acts like a bridge between p and r. Since each atomic proposition is linked up properly, no wonder the conclusion will follow.

$$
\begin{array}{l}
p \to q \\
\quad \underline{q \to r} \\
\therefore p \to r
\end{array}
$$

This argument form is called a **chain argument**. It is also a form of **pure hypothetical syllogism**. It is syllogism because it has exactly three propositions in the argument (two premises and one conclusion). It is hypothetical because it involves the use of material implication. It is pure because it only involves the use of material implication.

Another example of this argument form:

> Example (55): If you are human, you have a sense of morality. If you have a sense of morality, you will not kill. Therefore, if you are human, you will not kill.

We have already discussed other forms of hypothetical syllogism. (50)–(53) are **mixed hypothetical syllogisms.** Mixed because they involve some conditionals and some non-conditional statements. Hypothetical syllogisms concern conditionals or implications There are also conjunctive inference and disjunctive syllogisms.

Conjunctive inference has two types: composition and decomposition. Both are valid argument forms.

▶ Composition conjunctive inference

p	He is tall.
$\underline{q}$	He is handsome.
$\therefore p \,\&\, q$	Therefore, he is tall and handsome.

The proof should be rather straightforward.

p	q	$p \,\&\, q$
T	T	T
T	F	F
F	T	T
F	F	T

There is only one case in which all premises are true, i.e. the first row, and the conclusion is true in that case. That is, it is impossible for the conclusion to be false when all premises are true. Hence, the argument is valid.

▶ Decomposition conjunctive inference

$\underline{p \,\&\, q}$	He is tall and handsome.
$\therefore p$	Therefore, he is tall.

Or:

$\underline{p \,\&\, q}$	He is tall and handsome.
$\therefore q$	Therefore, he is handsome.

p	q	$p \& q$	p
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

p	q	$p \& q$	q
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

'$p \& q$' is true in only one case, namely when both conjuncts are true. Hence, given that $(p \& q)$ is true, p must be true, and q must be true as well.

There are also two common forms of **disjunctive syllogism.** One is valid while the other is invalid.

a $p \vee q$

$\underline{\neg p}$

$\therefore q$

Example: George is kind at heart or he has an ulterior motive. George is not kind at heart. Therefore, George has an ulterior motive.

Contrast with the following:

b $p \vee q$

$\underline{p}$

$\therefore \neg q$

Example: George is kind at heart or he has an ulterior motive. George is kind at heart. Therefore, George does not have an ulterior motive.

Which of the above argument forms is valid and which is invalid? Sometimes people can act for multiple reasons. Being kind-hearted and having an ulterior motive are not mutually exclusive. For a disjunction to be true, only one of the disjuncts needs to be true. Hence, if one of them is false, it must be the case that the other one is true, otherwise the disjunction does not hold. Therefore, (a) is a valid argument form. However, conversely, if one of them is true, it does not tell us whether the other is true or false. This is because whether or not the other proposition is true, the disjunction is true anyway. Therefore, (b) is invalid. We can prove the cases with truth-tables.

p	q	$p \vee q$	$\neg p$	q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

In the case of (a), illustrated above, there is only one case in which both premises are true, that is the third row. In this case, the conclusion is true. There are no other cases in which all premises are true. Therefore, the argument is valid.

p	q	$p \vee q$	p	$\neg q$
T	T	T	T	F
T	F	T	T	T
F	T	T	F	F
F	F	F	F	T

In the case of (b), all premises are true in the first and the second rows. However, the conclusion is false in the first row. Since it is possible for this argument form to have all true premises but a false conclusion, this argument form is invalid.

We can sum up the common valid and invalid argument forms as follows:

Key idea: Valid and invalid argument forms

Some common *valid* argument forms

Mixed hypothetical syllogism:

- $p \rightarrow q, p, \therefore q$ *(modus ponens)*
- $p \rightarrow q, \neg q, \therefore \neg p$ *(modus tollens)*

Pure hypothetical syllogism (chain argument):

- $p \rightarrow q, q \rightarrow r, \therefore p \rightarrow r$

Conjunctive inference:

- $p, q, \therefore p \,\&\, q$
- $p \,\&\, q, \therefore p$
- $p \,\&\, q, \therefore q$

Disjunctive syllogism:

- $p \vee q, \neg p, \therefore q$

Some common *invalid* argument forms

Fallacy of affirming the consequent:

- $p \rightarrow q, q, \therefore p$

Fallacy of denying the antecedent:

- $p \rightarrow q, \neg p, \therefore \neg q$

Fallacy in disjunctive syllogism:

- $p \vee q, p, \therefore \neg q$

Test your understanding by doing an exercise. You may also regard the exercise questions as more examples of common valid argument forms and fallacies.

Exercise 5.3: Common valid argument forms and fallacies

Identify the common argument forms in the following arguments and determine whether the arguments are valid *without* using the truth-table. Name the fallacy committed if there is a name for it.

1 If it rains, then the ground is wet. It rains. Therefore, the ground is wet.
2 If it rains, then the ground is wet. The ground is wet. Therefore, it rains.
3 If it rains, then the ground is wet. The ground is not wet. Therefore, it does not rain.
4 If it rains, then the ground is wet. It does not rain. So, the ground is not wet.
5 If the storm comes, then the river is flooded. If the river is flooded, then the houses are destroyed. Therefore, if the storm comes, the houses are destroyed.
6 Either Tom was naughty to play in mud or he fell into it by accident. Tom was not naughty. Therefore, he must have fallen into it by accident.
7 Either Tom was naughty to play in mud or he fell into it by accident. Tom was naughty. Therefore, he did not fall into the mud by accident.

MORE VALIDITY TESTING

Arguments come in many forms. Surely, not all arguments belong to the common argument forms or fallacies mentioned above. The truth-table method is a general method to test validity though. The examples below illustrate how to apply the method to handle arguments of any form.

> Example: (56): If China does not improve its human rights record, then it will receive continual scrutiny from the international community. Therefore, if China improves its human rights record, then it will not receive continual scrutiny from the international community.

Let p = China improves its human rights record, q = China receives continual scrutiny from the international community.

$\underline{\neg p \rightarrow q}$

$\therefore p \rightarrow \neg q$

p	q	$\neg p \rightarrow q$	$p \rightarrow \neg q$
T	T	F T $\underline{\text{T}}$ T	T $\underline{\text{F}}$ F T
T	F	F T T F	T T T F
F	T	T F T T	F T F T
F	F	T F F F	F T T F

There are three possible truth-value assignments under which the premise is true, namely, the first, the second and the third rows. The conclusion is false in the first row. Hence it is possible for the premise to be true yet the conclusion be false. The argument is invalid.

We can easily see why the argument is invalid. The premise expresses a necessary condition for China not receiving continual scrutiny from the international community, yet the conclusion asserts for a sufficient condition. The truth of the premise thus does not guarantee that of the conclusion.

Example (57): If he's in his room, then either he's working or he's up to something. He's in his room and he's not working. Therefore, he's up to something.

This example alerts us of the use of brackets. In the first sentence, 'either he's working or he's up to something' is the whole consequent to 'he's in his room', so a bracket is necessary to distinguish it from having only 'he's working' as the consequent. The argument form should be as follows.

Let p = he is in his room, q = he is working, r = he is up to something

$p \rightarrow (q \vee r)$

$\underline{p \ \& \neg q}$

$\therefore r$

There are three atomic propositions. So the truth-table needs eight rows.

p	q	r	$p \rightarrow (q \vee r)$	$p \,\&\, \neg q$	r
T	T	T	TT TTT	TFFT	T
T	T	F	TT TTF	TFFT	F
T	F	T	TT FTT	TTTF	T
T	F	F	TF FFF	TTTF	F
F	T	T	FT TTT	FFFT	T
F	T	F	FT TTF	FFFT	F
F	F	T	FT FTT	FFTF	T
F	F	F	FT FFF	FFTF	F

There is only one case in which the premises are true and that is the third row. In that case, the conclusion is also true. It is not possible for the premises to be true while the conclusion is false. Hence, the argument is valid.

> Example (58): If China extends its military influence over the South China Sea islands, then many Asian countries will protest and the USA will intervene. Yet only if many Asian countries protest will the USA intervene. Therefore, unless China extends its military influence over the South China Sea islands, the USA will not intervene.

This example has two phrases which we need to pay particular attention, to 'only if' in the second sentence and 'unless' in the final sentence, which is also the conclusion. We also need to be aware of the use of brackets in the first sentence.

> Suppose p = China extends its military influence over the South China Sea islands, q = many Asian countries protest, r = the USA intervenes

$p \rightarrow (q \,\&\, r)$

$\underline{r \rightarrow q}$ (or, $\neg q \rightarrow r$)

$\therefore p \vee \neg r$ (or, $\neg p \rightarrow \neg r, r \rightarrow p$)

In the first sentence, which is also a premise, both the protest from the many Asian countries and the intervention of the USA are the consequents of China's extension of military influence. Hence, a pair of brackets is needed to apply to the conjunction of the two results.

In the second sentence, 'only if' indicates that the protest is the necessary condition for American intervention. Hence, if there is no protest, there is no intervention. Or, taken in another way, if there is intervention, there must have been protests from the Asian countries.

In the last sentence, 'unless' works just like a disjunction. China's extension of influence is the necessary condition for American intervention. Thus, the last sentence is true in the following cases: if the USA intervenes, then it must be the case that China has extended its influence. Or, if China does not extend its influence, then the USA will not intervene. Thus, the following symbolizations are all correct: $(p \vee \neg r)$, $(\neg p \rightarrow \neg r)$, $(r \rightarrow p)$.

To test validity, we construct a truth-table for the argument:

p	q	r	$p \rightarrow (q \,\&\, r)$	$r \rightarrow q$	$p \vee \neg r$
T	T	T	TT TTT	TTT	TTFT
T	T	F	TF TFF	FTT	TTTF
T	F	T	TF FFT	TFF	TTFT
T	F	F	TF FFF	FTF	TTTF
F	T	T	FT TTT	TTT	FFFT
F	T	F	FT TFF	FTT	FTTF
F	F	T	FT FFT	TFF	FFFT
F	F	F	FT FFF	FTF	FTTF

In the fifth row, the premises are all true but the conclusion is false. It shows that there is no guarantee that the conclusion is true when all the premises are true. Hence, the argument is invalid. Although there are also cases in which all premises are

true and the conclusion is true (the first, the sixth and the eight rows), merely one case of false conclusion when all premises are true is enough to render the argument invalid.

> Example (59): If China extends its military influence over the South China Sea islands, many Asian countries will protest and the USA will intervene. Yet only if many Asian countries protest will the USA intervene. Therefore, unless China gives up extending its military influence over the South China Sea islands, the USA will not stop from intervening.

(59) has the same premises as (58); however, it has a different conclusion. So (59) is a different argument from (58). The conclusion of (59) involves some double negatives too. 'Gives up extending' does not contain negative words like 'not' or 'no'; however, the meaning is indeed negative. When one gives up something it means one would not do something. So the proper translation should be a negation of the proposition about extending influence. 'Not stop from intervening' is a double negative involving 'not' and 'stop', as the latter is negative in meaning. Classical logic assumes propositions are either true or false. Negating a proposition gets to its opposite truth-value. Negating twice would thus be back to its original truth-value. An axiom in a logical system is a formula that is always true. The following is an **axiom about double negation.**

$$\neg \neg p \equiv p$$

Thus, 'the USA will not stop from intervening' is logically equivalent to 'the USA will intervene'.

Using the same set of symbols, the argument should now be symbolized as follows.

$p \rightarrow (q \,\&\, r)$

$\underline{r \rightarrow q}$ or, $(\neg q \rightarrow \neg r)$

$\therefore \neg p \vee r$ or, $(p \rightarrow r, \neg r \rightarrow \neg p)$

p	q	R	$p \to (q \& r)$	$r \to q$	$\neg p \vee r$
T	T	T	TT TTT	TT T	FTTT
T	T	F	TF TFF	FT T	FTFF
T	F	T	TF FFT	TF F	FTTT
T	F	F	TF FFF	FT F	FTFF
F	T	T	FT TTT	TT T	TFTT
F	T	F	FT TFF	FT T	TFTF
F	F	T	FT FFT	TF F	TFTT
F	F	F	FT FFF	FT F	TFTF

Now in this argument, every time all the premises are true (the first, fifth, sixth and eighth rows), the conclusion is true. The argument is valid.

Below let's practise with an example involving equivalence.

Example (60): John goes to the cinema if and only if he has money. He has money if he works hard. But if he works hard, he cannot go to the cinema. So, he works hard, or he has the money but still does not go to the cinema

Let p = John goes to the cinema, q = John has money, r = John works hard

$p \equiv q$

$r \to q$

$\underline{r \to \neg p}$

$\therefore r \vee (q \& \neg p)$

It is also possible to conjunct the second and the third premises. The text does have the connective 'but', so it is alright to do so. Even if there is no 'but', it is still alright to conjoin premises and this yields no difference in truth-values. It is because, in fact, any argument can be taken as a complex proposition by itself: all premises conjunct to materially imply the conclusion.

Note that when we conjunct the second and the third premises in this example, we should add brackets first before conjoining. The argument is thus as follows.

$p \equiv q$

$\underline{(r \rightarrow q) \;\&\; (r \rightarrow \neg p)}$

$\therefore r \vee (q \;\&\; \neg p)$

p	q	r	$p \equiv q$	$r \rightarrow q$	$r \rightarrow \neg p$	$r \vee (q \;\&\; \neg p)$
T	T	T	TTT	TTT	TFFT	TT TFFT
T	T	F	TTT	FTT	FTFT	FF TFFT
T	F	T	TFF	TFF	TFFT	TT FFFT
T	F	F	TFF	FTF	FTFT	FF FFFT
F	T	T	FFT	TTT	TTTF	TT TTTF
F	T	F	FFT	FTT	FTTF	FT TTTF
F	F	T	FTF	TFF	TTTF	TT FFTF
F	F	F	FTF	FTF	FTTF	FF FFTF

In the second and eighth rows, all the premises are true but the conclusion is false. Hence, the argument is invalid. Indeed, just one such case is sufficient to conclude that the argument is invalid.

Example (61): I won't get drunk if I drink wine and spirits. It is because if I drink wine but not spirits, I will not get drunk. If I drink spirits but not wine, I will not get drunk either.

Unlike all the other examples, this example has the conclusion stated in the first sentence. We should be aware of the basic skills in identifying arguments and remember to put the conclusion in its proper place when we formulate the argument in its proper form.

Let p = I drink wine, q = I drink spirits, r = I get drunk

$(p \;\&\; \neg q) \rightarrow \neg r$

$\underline{(q \;\&\; \neg p) \rightarrow \neg r}$

$\therefore (p \;\&\; q) \rightarrow \neg r$

p	q	r	$(p \& \neg q) \to \neg r$	$(q \& \neg p) \to \neg r$	$(p \& q) \to \neg r$
T	T	T	T F F T T F T	T F F T T F T	T T T F F T
T	T	F	T F F T T T F	T F F T T T F	T T T T T F
T	F	T	T T T F F F T	F F F T T F T	T F F T F T
T	F	F	T T T F T T F	F F F T T T F	T F F T T F
F	T	T	F F F T T F T	T T T F F F T	F F T T F T
F	T	F	F F F T T T F	T T T F T T F	F F T T T F
F	F	T	F F T F T F T	F F T F T F T	F F F T F T
F	F	F	F F T F T T F	F F T F T T F	F F F T T F

There is only one case in which the premises are true but the conclusion is false and that is the first row. One case is enough to make sure that the conclusion does not always follow from the premises. Hence, the argument is invalid.

You may wonder so far that our examples all have cases where all the premises are true; all we do is to see whether the conclusion is always true in those cases. However, what if the premises cannot all be true, how would validity be determined in such a case? The answer is that in cases where the premises cannot be true, the argument is valid. We regard it as *vacuously* valid but it is valid nonetheless. It is valid because it is not possible for the conclusion to be false while the premises are true. When it is impossible to get all true premises, it is also impossible to get a false conclusion from them.

When it is impossible for all the premises to be true, we call the argument inconsistent. **Inconsistence** means that the propositions cannot be true together. Here is an example.

> Example (62): If the Bible is true, then the world was created by God. If the Big Bang theory is true, then the world was not created by God. The world was created and not created by God. Therefore, the Bible and the Big Bang theory are both true.
>
> Let p = the Bible is true, q = the Big Bang theory is true, r = the world was created by God
>
> $p \to r$
>
> $q \to \neg r$

$$\frac{r \,\&\, \neg r}{\therefore p \,\&\, q}$$

p	q	r	$p \to r$	$q \to \neg r$	$r \,\&\, \neg r$	$p \,\&\, q$
T	T	T	T T T	T F F T	T F F T	T T T
T	T	F	T F F	T T T F	F F T F	T T T
T	F	T	T T T	F T F T	T F F T	T F F
T	F	F	T F F	F T T F	F F T F	T F F
F	T	T	F T T	T F F T	T F F T	F F T
F	T	F	F T F	T T T F	F F T F	F F T
F	F	T	F T T	F T F T	T F F T	F F F
F	F	F	F T F	F T T F	F F T F	F F F

There is no case in which all the premises are true. Indeed, in this example, under no circumstance would one of the premises, '$r \,\&\, \neg r$', be true at all. Hence, it is not possible for all the premises to be true together. Yet, it is also impossible for all the premises to be true while the conclusion is false. Therefore, the argument is (vacuously) valid.

We may feel uncomfortable with the argument. Whether or not the argument is valid, there seems to be something wrong with the premises. Indeed. The problem is that we know intuitively something cannot be the case and not the case at the same time. The presence of a logical contradiction seems to us a sure sign of something wrong with the argument that we would rather accept this absurdity as evidence for the falsehood of some of the premises or the conclusion. Thus, in our mind, we probably would reconstruct the argument to make something sensible out of it. For instance:

> Example (63): If the Bible is true, then the world was created by God. If the Big Bang theory is true, then the world was not created by God. It cannot be the case that the world was both created and not created by God. Therefore, it cannot be the case that the Bible and the Big Bang theory are both true.

The logical form of this argument is:

$p \rightarrow r$

$q \rightarrow \neg r$

$\underline{\neg (r \,\&\, \neg r)}$

$\therefore \neg (p \,\&\, q)$

Note that in this argument, it is possible for the premises to be true together (the third, and the fifth up to the eighth rows) and in each case the conclusion is true. Hence this is a valid argument.

p	q	r	$p \rightarrow r$	$q \rightarrow \neg r$	$\neg (r \,\&\, \neg r)$	$\neg (p \,\&\, q)$
T	T	T	TTT	TFFT	TTFFT	FT TT
T	T	F	TFF	TTTF	TFFTF	FT TT
T	F	T	TTT	FTFT	TTFFT	TT FF
T	F	F	TFF	FTTF	TFFTF	TT FF
F	T	T	FTT	TFFT	TTFFT	TF FT
F	T	F	FTF	TTTF	TFFTF	TF FT
F	F	T	FTT	FTFT	TTFFT	TF FF
F	F	F	FTF	FTTF	TFFTF	TF FF

We do not feel uncomfortable with the argument in (63) because it is not a vacuously valid argument. Rather, it informs us of something: we are now certain that the Bible and the Big Bang theory cannot both be true if the premises are true. If someone wants to defend a different position that the Bible and the Big Bang theory are compatible, then they would have to deny some of the premises. The first premise seems uncontroversial because it is an orthodox position held by the Christian Church. So their strategy is probably to reject the Big Bang theory as being incompatible with the creation story.

Indeed, the above thinking process also mirrors exactly a reasoning strategy called argument by **reduction to absurdity** (in Latin, *reductio ad absurdum*). This strategy tells us to construct an argument in such a way that it leads to a logical contradiction. From this, we infer that at least one of the premises of this argument is false. The short truth-table method to be introduced in the next section makes use of this strategy. We shall discuss it further there.

To summarize, we have introduced the truth-table method for testing validity in this section. A truth-table is a complete list of all possible truth-value assignments of a proposition. We list all possible truth-value assignments of propositions in an argument, then proceed to check whether it is possible to obtain a case in which all the premises are true while the conclusion is false. If it is impossible, then the argument is valid. If it is possible, then the argument is invalid. The truth-table method is a general method for checking validity. There are some common argument forms already known to be valid or invalid. By identifying these argument forms, we can easily determine the argument's validity without drawing a truth-table, though of course, such proofs are available if we wish to draw their truth-tables.

The truth-table method works because a truth-table is a *complete and exhaustive* device to show all possible truth-value combinations. This is reminiscent of the rationale for the Venn diagram method in categorical logic. In categorical logic, the method works because a Venn diagram crafts the logical space into a number of areas *exhaustively* and reveals the *complete* set of possibilities for each area to have an object in it. Thus, everything is *transparent* to the mind. Nothing is hidden and nothing escapes attention as long as an agent is cautious enough to run or check a test. There is no hidden agenda, and there is no need for any mysterious cognitive faculties like speculation, inspiration, revelation or premonition. Moreover, the truth-table method is effective, in that every argument can be determined, within a definite number of steps, to be either valid or invalid. It does not depend on any contingent factors such as the intelligence, epistemic status or mental state of an agent, or the availability of information, strategy, etc. As we shall see in Section 5.7, some other methods of proof are not like that.

Key idea: Did you know?

The truth-table method was invented by Ludwig Wittgenstein in his early work, *Tractatus Logico-Philosophicus*.

> 'Philosophy simply puts everything before us, and neither explains nor deduces anything – since everything lies open to view there is nothing to explain. For what is hidden, for example, is of no interest to us.' (Ludwig Wittgenstein (1958), *Philosophical Investigations*, section 126)

The above quotation explains the reason for developing *a mechanism or an algorithm* to model the reasoning process. Indeed, it would not matter whether the process is carried out by human beings, other species, or even a machine. What is valid or invalid does not depend on human deliberation but on the reasoning pattern. Intelligence is but a matter of computation!

Some philosophers have developed *computational theories of mind* inspired by this way of thinking about reasoning. According to this view, a mind is nothing more than a computer. Mental phenomena are identified by functions. It would not matter whether a mind is composed of organic wet matter, silicon, or any physical matter *per se*, as long as it functions. Further exploration of this topic is beyond the scope of this book, so let's move on to an exercise to practise what we've learned so far.

Exercise 5.4: The truth-table method

Symbolize the following arguments and test their validity with truth-tables. Remember to state what your symbols represent.

1 It is not the case that she either forgot or wasn't able to finish. Therefore, she was able to finish.

2 Either the manager didn't notice the change or else he approves of it. He noticed it all right. So he must approve of it.

3 If humans have free will, then God cannot know what every human being will do next. But there is no free will; therefore, God knows what each one of us will do next.

4 If God exists, then he must be good. If God is good, then he cannot tolerate evil. But we see evil every day. Therefore, God does not exist.

5 If the MP votes against this bill, then he is opposed to penalties against tax evaders. Also, if the MP is a tax evader himself,

then he is opposed to penalties against tax evaders. Therefore, if the MP votes against this bill, he is a tax evader himself.

6 It is not fair to smoke around non-smokers if second-hand cigarette smoke really is harmful. Second-hand smoke is harmful or the Lung Association would not be telling us that it is. It thus follows that if they are telling us that it's harmful, then it is enough to conclude that it's not fair to smoke around non-smokers, as second-hand cigarette smoke is harmful.

7 Only if she respects her friends as individuals can she have many friends. If she respects them as individuals, then she cannot expect them all to behave alike. She does have many friends. Therefore, she does not expect them all to behave alike.

8 If equality of opportunity is to be achieved, then those people previously disadvantaged should now be given special opportunities. If those people previously disadvantaged should now be given special opportunities, then some people receive preferential treatment. If some people receive preferential treatment, then equality of opportunity is not to be achieved. Therefore, equality of opportunity is not to be achieved.

9 If people are entirely rational, then either all of a person's actions can be predicted in advance or the universe is essentially deterministic. Not all of a person's actions can be predicted in advance. Thus, if the universe is not essentially deterministic, then people are not entirely rational.

10 The more we learn, the more we know. But only if we know can we forget. And the more we forget, the less we learn. So why learn?

5.6 The short truth-table method

Although the truth-table method is justified and is fully decidable (i.e. always yields a result), it requires quite a lot of computation. This may prove *inefficient* particularly when an argument is long or involves many atomic propositions. The number of rows required in a truth-table is exponentially dependent on the number of atomic propositions. Yet human beings have limited cognitive capacities. A more efficient method is desired. Hence, we introduce the short truth-table method here.

The short truth-table method makes use of two ideas. First, validity can be defined as always getting a true conclusion from true premises, or equivalently, as the impossibility of getting a false conclusion from all true premises. It is a lot easier to test the latter than the former because it only requires one counter-example. If we can quickly identify that counter-example, then there is no need to enumerate all truth-value assignments on an argument. This will save a lot of time.

Second, we can apply the reasoning strategy of *reductio ad absurdum* to help us. We mentioned that strategy before. Indeed, we use *reductio* all the time in real life. Consider the following scenario. Suppose you are a detective for this case and you find the following.

> Example (64): The caddy in the kitchen was full of sweets last night at 11pm. But it is empty now. Susan was at a school camp for two days. Can Susan be the thief?

You probably would say that the answer is no. But why? The reasoning process runs as follows.

> (64') (a) Assume Susan is the thief.
>
> (b) If Susan is the thief, then she was in the kitchen last night after 11pm.
>
> (c) Therefore, Susan was in the kitchen last night after 11pm.
>
> (d) However, Susan was at the school camp last night after 11pm.
>
> (e) Susan cannot be in the kitchen and at the school camp at the same time.
>
> (f) Therefore, Susan was not in the kitchen last night after 11pm.
>
> (g) Therefore, Susan is not the thief.

Let p = Susan is the thief, q = Susan was in the kitchen last night after 11pm. The above reasoning can be formulated as follows.

p (Assumption: this is our hypothesis)

$\underline{p \rightarrow q}$

$\therefore q$ (*Modus ponens*)

$\neg q$ (Assumption: it is a given fact)

$\underline{q \& \neg q}$

$\therefore \neg p$

Reductio ad absurdum works by making an assumption and then trying to prove it wrong, because if it were true, it would lead to a contradiction (i.e. an absurdity). Apply the strategy to our aim of finding a way to test invalidity. Let us suppose an argument is invalid and assign truth-values to its component atomic propositions. If we find out that it leads to a contradiction, then we are justified in asserting that our assumption must be wrong, namely, the argument is not invalid. An argument is either valid or invalid. Hence, an argument not invalid is a valid argument We thus prove that the argument is valid. If we can consistently assign a set of truth-values under the assumption so that it does not lead to any contradiction, then it means the assumption is true indeed. The argument is thus invalid. In short, we prove validity by working out the exact truth-value assignment under which the assumption of invalidity is fulfilled.

To execute the idea, we need to remind ourselves of the definite truth-value assignments in each logical connective. These situations help us determine the truth-values of some atomic propositions with certainty.

- Negation always yields an opposite truth-value.
- A conjunction is true only when both conjuncts are true.
- A disjunction is false only when both disjuncts are false.
- A material implication is false only when the antecedent is true but the consequent is false.
- An equivalence relation is true if and only if both sides have the same truth-values. It is false in all other cases.

Let us illustrate with some examples. The methods for symbolizing an argument are the same as before, so we will work directly on symbolized argument forms.

Example (65): $p \rightarrow q, r \rightarrow q \ / \therefore r \rightarrow p$

Put the above argument into a truth-table. There are three atomic propositions, so there are three columns on the left to signal the truth-value of each atomic proposition. Then each premise and conclusion occupies a column. We only need one row though because our aim is just to find out a truth-value assignment which makes all the premises true but the conclusion false.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$

Assume the argument is invalid; that is, all the premises are true and the conclusion is false. We put the truth-values in to the truth-table under each respective proposition. Then we work *backwards* to get a truth-value assignment of p, q and r that makes the argument invalid. Remember that all truth-values have to be placed precisely beneath the operation they relate to. So in this case, we put the truth-values under neath the arrows.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
			T	T	F

There are many combinations that make a material implication true, so we cannot get definite truth-values of its components in such cases. However, there is only one combination that makes a material implication false. Therefore, we can start from there and attribute truth-values to the false implication. $r \rightarrow p$ is false; thus, we determine that r is true and p is false.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
			T	T	T F F

Put the result to other propositions and see if it makes a consistent assignment.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
			T	T T	T F F

If r is true and $(r \rightarrow q)$is true, then q must be true because if q is false, then$(r \rightarrow q)$ is false. So we find a definite truth-value for q, which is T.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
			T	T T T	T F F

Now apply the result that p is F and q is T to the remaining proposition.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
F	T	T	F T T	T T T	T F F

It works! When p is false and q is true, then $(p \rightarrow q)$ is true, just the result needed. So, we do not have a contradiction. We can consistently assign p to be F, q to be T and r to be T and get the result that all the premises are true but the conclusion is false. Since it is possible that all the premises are true but the conclusion is false, our initial assumption is true indeed. The argument is invalid.

We can compare this result with that generated using the (full) truth-table method.

p	q	r	$p \rightarrow q$	$r \rightarrow q$	$r \rightarrow p$
T	T	T	T T T	T T T	T T T
T	T	F	T T T	F T T	F T T
T	F	T	T F F	T F F	T T T
T	F	F	T F F	F T F	F T T
F	T	T	F T T	T T T	T F F
F	T	F	F T T	F T T	F T F
F	F	T	F T F	T F F	T F F
F	F	F	F T F	F T F	F T F

What we get in the short truth-table method is identical to the fifth row of the (full) truth-table. There are other rows which show true premises and a true conclusion. However, they are not relevant because no matter how many other assignments in which the conclusion follows the premises, one counter-example is sufficient to establish the invalidity of an argument. We just need to identify that instance. Using the short truth-table

method, we save ourselves a lot of trouble by drawing only one row of the truth-table instead of eight.

If we become sufficiently familiar with the short truth-table method, we can even skip the few columns for truth-value assignments of the atomic propositions and work directly on the premises and conclusion only.

$p \to q$	$r \to q$	$r \to p$
F T T	T T T	T F F

The short truth-table method requires familiarity with the truth-value assignments of all truth-functional connectives. It also needs the agent to reason backwards from truth-values of complex propositions to those of atomic ones. If you are sufficiently confident with the operations, you may use this method and enjoy getting a speedy solution. This method feels more intelligent because actual reasoning hardly follows mechanical computations. However, not all arguments are readily solvable by this method. Sometimes there are cases when no definite assignment is appropriate. In such cases, we may need to construct the full truth-table.

Example (66): $(p \vee q) \to r, q \,\&\, \neg r \;\therefore \neg p$

Assume this is an invalid argument:

$(p \vee q) \to r$	$q \,\&\, \neg r$	$\neg p$
T	T	F

Negation always yields a definite truth-value – just the opposite of the original one. So we should start assigning values in this case to the proposition, $\neg p$. $\neg p$ is false, so p must be true. Moreover, a conjunction is true only if both conjuncts are true. $(q \,\&\, \neg r)$ is true, so we can infer that both q and $\neg r$ are true. $\neg r$ is true implies that r is false.

$(p \vee q) \to r$	$q \,\&\, \neg r$	$\neg p$
T	T T T F	F T

Put all values to the remaining proposition and see if they yield a consistent assignment. r is false, according to the second premise.

When a consequent is false, the only possibility for a material implication to be true is that the antecedent is false. Hence, ($p \vee q$) has to be false. For a disjunction to be false, both disjuncts must be false. However, p is true according to the conclusion and q is also true from the inference in the second premise, so both disjuncts are true. Here, we have found two contradictions. Yet indeed one contradiction is enough to render the assumption false. We underline the contradiction(s). The assumption must therefore be mistaken and the argument is valid.

$(p \vee q) \to r$	$q \,\&\, \neg r$	$\neg p$
$\underline{F}$ F $\underline{F}$ T F	$\underline{T}$ T T F	F $\underline{T}$

Example (67): $p \vee \neg q, (r \equiv \neg p) \to \neg q \;/\therefore q \to \neg r$

Assume the argument is invalid.

$p \vee \neg q$	$(r \equiv \neg p) \to \neg q$	$q \to \neg r$
T	T	F

There are multiple possible truth-value assignments when a disjunction is true or a material implication is true. So, we cannot start from the first two propositions. But when a material implication is false, we get a definite pair of truth-values: the antecedent is true and the consequent is false. So in the conclusion, we get q to be true and $\neg r$ false. $\neg r$ is false, so r is true. Now we put these values back to the other propositions. When q is true, $\neg q$ is false. Yet for a disjunction to be true, at least one of the disjuncts must be true. $\neg q$ is false, so p must be true. In the second premise, when $\neg q$ is false and the implication is true, the antecedent must be false. So, ($r \equiv \neg p$) is false. Yet r is true from the conclusion. For an equivalence relation to be false, the components must have different truth-values. So $\neg p$ has to be false and p is true.

$p \vee \neg q$	$(r \equiv \neg p) \to \neg q$	$q \to \neg r$
TT F T	T F FT T F T	T F FT

We now have a consistent truth-value assignment: p is true, q is true and r is true. No contradiction is formed. Hence, the assumption is correct. The argument is invalid.

Example (68): $(p \to q) \vee (r \to q), p \,\&\, (p \to \neg q) \;/ \therefore r$

Assume the argument is invalid. That is, all premises are true and the conclusion is false.

$(p \to q) \vee (r \to q)$	$p \,\&\, (p \to \neg q)$	r
T	T	F

The second premise is true, so the two conjuncts must both be true. p is true and $(p \to \neg q)$ is true. Since p is true and $(p \to \neg q)$ is true, $\neg q$ must be true. $\neg q$ is true, so q is false. Apply these results to the first premise, $(p \to q)$ is false. A true disjunction must have at least one true disjunct. Since $(p \to q)$ is false, $(r \to q)$ must be true. From the conclusion, it is known that r is false. A false antecedent always gives a true implication. So even when q is false, $(r \to q)$ is true. No contradiction is found. Therefore, the argument is invalid.

$(p \to q) \vee (r \to q)$	$p \,\&\, (p \to \neg q)$	r
T F F T F T F	T T T T T F	F

In the next two examples, we will also discuss another type of argument called the dilemma.

Let us tell a story first. Once upon a time, there was a lawsuit between an Ancient Greek lawyer and his favourite student. The two of them had agreed that when the student graduated and won his first case, he had to pay the teacher for educating him so well. Whereas if the student lost his first case, that implied the teaching was not good and the student did not have to pay anything. When the student graduated, he filed his first court case by suing his teacher for asking him to pay. His reasoning ran as follows:

> Example (69): The student either wins or loses. If he wins, the court rules that the teacher cannot ask him to pay, so, he does not have to pay. If he loses, according to the agreement between the student and the teacher, since he loses his first case, he does not have to pay. In either case, he does not have to pay.

This seemingly put the student in a sure win position. However, the teacher was no weakling either. He defended as follows.

> (69') The student either wins or loses. If he wins, according to the contract between the student and the teacher, he

has to pay. If he loses, the court rules that the teacher can charge him, so, he has to pay. In either case, he has to pay!

Both arguments have the following argument form:

$$p \vee q, p \to r, q \to r /\therefore r$$

We can easily prove the validity of the argument form with a truth-table. Now, let us try the short truth-table method.

Assume the argument is invalid, all premises are true and the conclusion is false.

$p \vee q$	$p \to r$	$q \to r$	r
F T T	F T F	F T F	F

r is false in the conclusion. Apply this to the second premise. For a true material implication with a false consequent, the antecedent must be false. Hence, p is false. Similarly, when r is false and $(q \to r)$ is true, q must be false. Apply these results to the first premise, if p is false and its disjunction with q is true, then q must be true. This contradicts the previous result that q is false. The assumption is therefore mistaken. The argument is valid.

This argument form belongs to a general type of argument form called a **dilemma.** It has two horns: p and q, each entailing some results. Depending on how the argument goes, a dilemma is further distinguished into two types. A **constructive dilemma** argues from how the horns are related to how the results are related. (69) is a special case of a constructive dilemma in which both horns lead to the same result. The general form of a constructive dilemma is as follows:

$$p \vee q, p \to r, q \to s /\therefore r \vee s$$

Let us prove its validity. First, assume the argument is invalid:

$p \vee q$	$p \to r$	$q \to s$	$r \vee s$
F T T	F T F	F T F	F F F

In the conclusion, for a disjunction to be false, both disjuncts must be false. Hence, we get that r is false and s is false. Then in the second premise, for a material implication to be true and its

consequent false, the antecedent must be false. Hence, p is false. Similarly, in the third premise, we get that q is false. In the first premise, for a disjunction to be true, at least one of the disjuncts must be true. p is false, so q must be true. We thus arrive at a contradiction: q is both true and false. The assumption is mistaken and the argument is valid.

Another type of dilemma argues backwards from what the entailments are *not* to what the horns are not. This type of dilemma is called a **destructive dilemma**. It has the following general form:

$$p \rightarrow r, q \rightarrow s, \neg r \vee \neg s \, / \therefore \neg p \vee \neg q$$

Let us check its validity with a short truth-table.

$p \rightarrow r$	$q \rightarrow s$	$\neg r \vee \neg s$	$\neg p \vee \neg q$
T T T	T T $\underline{\text{T}}$	F T T T $\underline{\text{F}}$	F T F F T

Assume the argument is invalid. The conclusion is false and since it is a disjunction, both disjuncts must be false; i.e. both p and q are true. If p is true and the first premise is true, then r must be true. If q is true and the second premise is true, then s must be true. If r is true, then $\neg r$ is false. But the third premise is true, so one of the disjuncts must be true. $\neg s$ has to be true and s false. This contradicts the previous result that s is true. The assumption is mistaken and the argument is valid.

The following is a concrete example of a destructive dilemma:

> Example (70): I can earn money for myself or serve others. If I earn money for myself, I will be happy. If I serve others, others will be happy. It cannot be the case that both I and others are happy. Therefore, it cannot be the case that I earn money for myself and serve others.

Can you symbolize the argument? Let p = I earn money for myself, q = I serve others, r = I am happy, s = others are happy.

$$p \vee q, p \rightarrow r, q \rightarrow s, \neg (r \,\&\, s) \, / \therefore \neg (p \,\&\, q)$$

$\neg (r \,\&\, s)$ is equivalent to $(\neg r \vee \neg s)$. $\neg (p \,\&\, q)$ is also equivalent to $(\neg p \vee \neg q)$. So it is exactly in the form of the destructive dilemma. We have already proved the validity of this form. Now

of course, you can argue that although this is a valid argument, it is not a good argument. It is because it is unsound. At least some of its premises are arguably false, notably the premise that it can't be that case that both I and others are happy!

The short truth-table method does not always work. We sometimes fail to find a determinate truth-value assignment to prove whether or not the assumption is true. Our final example shows this.

Example (71): $p \to q, r \vee q, r \equiv s / \therefore r \& s$

Assume the argument is invalid.

$p \to q$	$r \vee q$	$r \equiv s$	$r \& s$
T	T	T	F

There are many combinations of truth-values for each of the propositions above. That means, we do not have a base to start with. We simply cannot go on but have to revert to using the full truth-table. Sometimes we may need to stop halfway through. So in this sense, the short truth-table method, though efficient, is not always effective.

Exercise 5.5: The short truth-table method

Test the following argument forms by using short truth-tables.

1 $(A \vee B) \to C$, $(A \& C) \equiv \neg B$, $\therefore B \to \neg C$
2 $(A \to B) \& [(A \& B) \to C]$, $A \to (C \to B)$, $\therefore A \to \neg B$
3 $(A \vee B) \to C$, $D \vee [C \to (\neg A \& \neg B)] / \therefore A \to (B \vee D)$
4 $(A \to B) \& (C \to \neg A)$, $A \vee C$, $\therefore \neg B \vee A$
5 $A \to (B \vee C)$, $(\neg B \to C) \& A$, $C \& \neg B$, $\therefore A \equiv (B \to C)$

5.7 Natural deduction

Let us review what we have learned so far. We have introduced five common logical connectives and their truth-tables. We have learned how to translate ordinary language arguments into their symbolized forms and how to test the validity of arguments using the (full or long) truth-table method as well as short truth-tables. Both methods make use of the same idea of a truth-table;

that is, to unroll all possible truth-value assignments so we can do things like checking validity. Indeed, both the truth-table method and the Venn diagram method in categorical logic share a common orientation: since everything lies open, completeness and exhaustiveness of the objective logical space is sought and is transparent to the mind.

Just as there is a rule method for categorical logic, there is also a rule method for propositional logic, which is called natural deduction. The rule methods and the methods introduced above (i.e. Venn diagrams and truth-tables) represent quite different ways of thinking. The table-and-diagram methods seek a model, a whole picture, an overview from no point of view; whereas the rule methods seek a way for an agent to navigate in. An agent would have to memorize and apply given normative rules. Often, justification of the rules themselves is left as a question separate to their application, whereas the justification of the table-and-diagram methods is intrinsic to the ideas of the methods themselves.

While the rule method of categorical logic only requires a few rules to be effective, the rule method of propositional logic is not as neat. More rules are needed. Moreover, given a certain number of rules, the natural deduction method is not always effective. Users need insights, intelligence and strategies in making moves. This is appealing in a way as it resembles the natural thinking process and natural deduction can be fast sometimes. However, it also means that logic is not easy to master as it is not a mechanical process at all. Both rule methods of categorical and propositional logics were developed since Ancient Greece.

Natural deduction has two sets of rules. One set involves known valid argument forms, including all those discussed in the previous sections. The other set involves truth-functional equivalences, to make transformation between propositions possible. The rule method in categorical logic only needs four rules, but there are roughly twenty rules in natural deduction, ten in each category. Let us list them all and then discuss the strategies for using them by looking at some examples. We need

to memorize all rules and their abbreviations so that w
quote them as justification in the proofs.

RULES OF INFERENCE

1 *Modus ponens (MP)*: $p \to q, p /\therefore q$

2 *Modus tollens* (*MT*): $p \to q, \neg q /\therefore \neg p$

3 Hypothetical syllogism (HS): $p \to q, q \to r /\therefore p \to r$

4 Disjunctive syllogism (DS): $p \vee q, \neg p /\therefore q$

5 Constructive dilemma (CD): $p \vee q, (p \to r) \,\&\, (q \to s) /\therefore r \vee s$

6 Destructive dilemma (DD): $p \to r, q \to s, \neg r \vee \neg s, \therefore \neg p \vee \neg q$

7 Simplification (Simp): $p \,\&\, q /\therefore p$

8 Conjunction (Conj): $p, q /\therefore p \,\&\, q$

9 Addition (Add): $p /\therefore p \vee q$

10 Absorption (Abs): $p \to q \ /\therefore p \to (p \,\&\, q)$

REPLACEMENT (OR, TRANSFORMATION) RULES:

1 Double negation (DN): $p \equiv \neg\neg p$

2 Commutation (Com): $(p \vee q) \equiv (q \vee p), (p \,\&\, q) \equiv (q \,\&\, p)$

3 Association (Assoc): $[p \vee (q \vee r)] \equiv [(p \vee q) \vee r], [p \,\&\, (q \,\&\, r)] \equiv [(p \,\&\, q) \,\&\, r]$

4 Distribution (Dist): $[p \,\&\, (q \vee r)] \equiv [(p \,\&\, q) \vee (p \,\&\, r)], [p \vee (q \,\&\, r)] \equiv [(p \vee q) \,\&\, (p \vee r)]$

5 De Morgan's laws (DeM): $\neg (p \,\&\, q) \equiv (\neg p \vee \neg q), \neg (p \vee q) \equiv (\neg p \,\&\, \neg q)]$

6 Transposition (Trans): $(p \to q) \equiv (\neg q \to \neg p)$

7 Material implication (Impl): $(p \to q) \equiv (\neg p \vee q)$

8 Material equivalence (Equiv): $(p \equiv q) \equiv [(p \to q) \,\&\, (q \to p)], (p \equiv q) \equiv [(p \,\&\, q) \vee (\neg p \,\&\, \neg q)]$

9 Exportation (Exp): $[(p \,\&\, q) \to r] \equiv [p \to (q \to r)]$

10 Tautology (Taut): $p \equiv (p \vee p), p \equiv (p \,\&\, p)$

Some of the rules are more intuitive than others. Yet, all of them can be tested with truth-tables. So we may regard the truth-table method as their ultimate justification. Taking these rules as given, I will illustrate below how to apply them to obtain a deduction.

A note about the format first. Normally we are given an argument to prove in a deduction. We number the premises and put the conclusion at the end of the last premise. We also number every step deduced from the premises, so each numbered line represents something that is obtained under the given premises. The goal is to work towards the conclusion. So at the end the conclusion requiring to be proved should appear in one of the numbered lines. Every step that we deduce from the premises has to be given a justification, normally by stating the numbers of the propositions and the relevant rules involved.

We almost always use the strategy of *working backwards* in deductions. That is, we look at the conclusion and work out how to connect it with the premises, rather than blindly developing the premises in all possible directions. Apart from this universal move, there are two more common strategies, namely, the **Conditional Proof** (CP) and the **Reduction to Absurdity** or the **Reduction to Contradiction** (Contra).

Now let us begin with some easy examples. For brevity, I will just use symbolized arguments and leave all matters of translation from ordinary language aside.

Example (72): $p \rightarrow q, q \rightarrow r, \neg r / \therefore \neg p$

First we write down the argument in the form of numbered statements.

1 $p \rightarrow q$

2 $q \rightarrow r$

3 $\neg r / \therefore \neg p$

We want to prove that $\neg p$. From the premises, p is only related to q. So to get to p, we need to get something about q. q is related to r and something about r is obtained from the premises. So it seems easy to see that lines 2 and 3 can form an inference to get something related to q, and then the result can

be used to deduce something about p. We write down thi
of reasoning and justifications as follows.

1	$p \to q$	
2	$q \to r$	
3	$\neg r \;/\; \therefore \neg p$	
4	$\neg q$	2, 3 MT
5	$\neg p$	1, 4 MT

By performing *modus tollens* between 2 and 3, we get 4. Then applying *modus tollens* again on 1 and 4, we get $\neg p$, which is our desired result. The argument is thus proved.

> Example (73): $p \to (q \to r), (\neg p \to t) \,\&\, (\neg q \to s), \neg t \,\&\, \neg s \;/\; \therefore r$

(73) looks complicated. It involves five atomic propositions. If this proof is done by computing a truth-table, it will need 32 rows, which is quite lengthy. It is more efficient to prove it using natural deduction.

Let's work backwards. We want to prove r and from the premises, r is related to p and q only. So we should look for something that will get us to p and q. In this case t and s are related to p and q, so we may use them.

1	$p \to (q \to r)$	
2	$(\neg p \to t) \,\&\, (\neg q \to s)$	
3	$\neg t \,\&\, \neg s \;/\; \therefore r$	
4	$\neg p \to t$	2 Simp
5	$\neg q \to s$	2 Simp
6	$\neg t$	3 Simp
7	$\neg s$	3 Simp
8	$\neg\neg p$	4, 6 MT
9	p	8 DN
10	$\neg\neg q$	5, 7 MT

11	q	10 DN
12	$q \to r$	1, 9 MP
13	r	12, 11 MP

Alternatively, we can apply the rule of exportation in the last few steps and write the proof slightly differently as follows.

1	$p \to (q \to r)$	
2	$(\neg p \to t) \,\&\, (\neg q \to s)$	
3	$\neg t \,\&\, \neg s \,/ \therefore \mathrm{r}$	
4	$\neg p \to t$	2 Simp
5	$\neg q \to s$	2 Simp
6	$\neg t$	3 Simp
7	$\neg s$	3 Simp
8	$\neg\neg p$	4, 6 MT
9	p	8 DN
10	$\neg\neg q$	5, 7 MT
11	q	10 DN
12	$p \,\&\, q$	9, 11 Conj
13	$(p \,\&\, q) \to r$	1 Exp
14	r	13, 12 MP

Both examples above involve tearing down complex propositions into atomic ones by MP or MT. Sometimes, however, we also need to deliberately construct a complex proposition to make the derivation work.

Example (74): $\neg (q \,\&\, r) \to p,\ s \to \neg q,\ s \,/ \therefore p$

Here in (74), we want to prove that p. We know from the first premise that if we have $\neg (q \,\&\, r)$, then by MP, we can get p. The second and third premise would get us $\neg q$. But we need $\neg (q \,\&\, r)$, not just $\neg q$. So we need to see if we can construct $\neg (q \,\&\, r)$ out of $\neg q$. To do so, we first need to dissolve the bracket. By DeMorgan's law, $\neg (q \,\&\, r)$ is equivalent to $\neg q \vee \neg r$. We have $\neg q$

already, so we just need to disjunct it with $\neg r$. $\neg r$ is not found anywhere in the other premises. However, fortunately, we know that adding any disjunct to a true proposition is true. $\neg q$ is true, so adding anything to it will give a true proposition. We can thus freely manipulate the situation to construct propositions to the shape we need.

1	$\neg (q \,\&\, r) \to p$	
2	$s \to \neg q$	
3	$s \,/ \therefore p$	
4	$\neg q$	2, 3 MP
5	$\neg q \vee \neg r$	4 Add
6	$\neg (q \,\&\, r)$	5 DeM
7	p	1, 6 MP

If the conclusion is a conditional argument, we have another strategy to employ which is called a **Conditional Proof (CP)**. The idea is simple. A conditional has an antecedent and a consequent. It does not claim that the antecedent is the case, but only that if it is, then the consequent follows. If we deliberately grant the antecedent and obtain the consequent somewhere in the deduction process, then the conditional relation is true. We just need to **discharge** the assumption: instead of assuming that the antecedent is the case, we say that *if* the antecedent is true, *then* the consequent is true, which is exactly the conclusion of the original argument. CP, in a nutshell, is to prove an argument by assuming the antecedent of the conclusion, working out its consequent and then discharging the assumption itself.

Remember, the antecedent added to the deduction is an extra assumption and not one of the original premises. All added assumptions *must* be discharged before we finish the proof. If there are several assumptions added, making loops within loops, then they must be discharged in exactly the reverse order so that every loop completes itself and does not depend on more assumptions than it really does. If these requirements are not fulfilled, then the original argument is not proved. To remind us that an assumption is merely our addition, we draw a line or a

half bracket on the left margin to indicate all deductions under the added assumption until it is discharged.

Example (75): $p \vee (q \rightarrow r), q \,/\, \therefore \neg p \rightarrow r$

The conclusion of (75) is a conditional, so we can try CP. Suppose we assume $\neg p$ is true. Then, by disjunctive syllogism, we get $q \rightarrow r$ from the first premise. With q and $q \rightarrow r$, we can get to r. That is, $\neg p$ leads to r. That is exactly the desired result needing to be proved in the original argument.

1	$p \vee (q \rightarrow r)$	
2	$q \,/\, \therefore \neg p \rightarrow r$	
3	$\neg p$	CP premise
4	$q \rightarrow r$	1, 3 DS
5	r	4, 2 MP
6	$\neg p \rightarrow r$	3–5 CP

Sometimes more than one CP premise is needed. In the following example (76), simply assuming the antecedent of the conclusion does not get us to the required consequent. Specifically, to get to t, we need $\neg q \rightarrow s$. To get $\neg q \rightarrow s$, we need another CP to assume $\neg q$, so it can lead to s.

Example (76): $p \rightarrow [q \vee (r \,\&\, s)], (\neg q \rightarrow s) \rightarrow t \,/\, \therefore p \rightarrow t$

1	$p \rightarrow [q \vee (r \,\&\, s)]$	
2	$(\neg q \rightarrow s) \rightarrow t \,/\, \therefore p \rightarrow t$	
3	p	CP premise
4	$q \vee (r \,\&\, s)$	1, 3 MP
5	$\neg q$	CP premise
6	$r \,\&\, s$	4, 5 DS
7	s	6 Simp
8	$\neg q \rightarrow s$	5–7 CP
9	t	2, 8 MP
10	$p \rightarrow t$	3–9 CP

Sometimes, the conclusion involves a conditional within a conditional. This is another case where we need multiple CPs. In (77), we cannot just assume p, for without u or v we cannot get to w and yet w is needed in the inner bracket of the conclusion. So we need to assume u as well.

Example (77): $p \to (q \,\&\, r), s \to t, q \to (r \to \neg t), (u \vee v) \to w$ / $\therefore p \to [u \to (w \,\&\, \neg s)]$

1	$p \to (q \,\&\, r)$	
2	$s \to t$	
3	$q \to (r \to \neg t)$	
4	$(u \vee v) \to w$ / $\therefore p \to [u \to (w \,\&\, \neg s)]$	
5	p	CP premise
6	u	CP premise
7	$u \vee v$	6 Add
8	w	4, 7 MP
9	$(q \,\&\, r) \to \neg t$	3 Exp
10	$q \,\&\, r$	1, 5 MP
11	$\neg t$	9, 10 MP
12	$\neg s$	2, 11 MT
13	$w \,\&\, \neg s$	8, 12 Conj
14	$u \to (w \,\&\, \neg s)$	6–13 CP
15	$p \to [u \to (w \,\&\, \neg s)]$	5–14 CP

Sometimes it is very hard to get to the conclusion and none of the strategies above is useful. In that case, we may try to force a proof using the strategy of reduction to absurdity (*reductio ad absurdum*), also called reduction to contradiction (*contra*). We have actually already used this strategy in the short truth-table method. The strategy works by assuming the negation of the conclusion to be proved in order to get a contradiction. If the assumption leads to a contradiction, then the assumption is false – the negation of the negated conclusion should be the case and by

double negation, the negation of a negated conclusion is just the conclusion itself. So we get our desired conclusion. The added assumption is also discharged at this point. Let us try an example.

Example (78): $p \vee q, p \rightarrow r, q \rightarrow \neg r, \neg (r \,\&\, \neg r) \,/\, \therefore \neg (p \,\&\, q)$

1	$p \vee q$	
2	$p \rightarrow r$	
3	$q \rightarrow \neg r$	
4	$\neg (r \,\&\, \neg r) \,/\, \therefore \neg (p \,\&\, q)$	
5	$p \,\&\, q$	Premise
6	p	5 Simp
7	r	2, 6 MP
8	q	5 Simp
9	$\neg r$	3, 8 MP
10	$r \,\&\, \neg r$	7, 9 Conj
11	$\neg (p \,\&\, q)$	5-10, 4 Contra

Using sufficient rules permitted and the strategies introduced above, natural deduction is effective, i.e. a proof should always be constructed if an argument is indeed valid. Propositional logic itself is sound and complete, so a method of proof would also be sound and complete. It is just a matter of how many rules are permitted: the more rules that are permitted, the more easily a proof is constructed. However, whether an individual agent can construct a proof by deduction is subject to epistemic factors, requiring introspection, rational intuition and also some cognitive capacities such as memory. So it may not suit all individuals. I thus recommend taking the truth-table method as the default because it is mechanical and always decidable.

A further remark before we close this section. A logical system is **sound** if and only if it does not allow a deduction beginning with true propositions to end up with false ones. A logical system is **complete** if and only if for every truth-functionally valid argument, there is a deduction in our system of rules that allows us to deduce

the conclusion of that argument from its premises. Logicians have conducted formal proofs of soundness and completeness of propositional logic; however, they are very technical and it is beyond the scope of this book to introduce them. Interested readers should consult other literature, such as Geoffrey Hunter's *Metalogic: An Introduction to Metatheory of Standard First Order Logic* (University of California Press, 1973).

5.8 Chapter summary

To summarize, propositional logic is a logic about truth-functional connections *between* atomic propositions. It takes the atomic proposition as the basic unit but does not analyse its internal logical structure. We looked at five common truth-functional connectives: negation, conjunction, disjunction, material implication and material equivalence. Each function is defined by its truth-functional relations, represented in a truth-table.

FIVE COMMON TRUTH-FUNCTIONAL CONNECTIVES

- **Negation** is a function which always give opposite truth-values.
- **Conjunction** is a function that is true only when both conjuncts are true.
- **Disjunction** is a function that is false only when both disjuncts are false.
- **Material implication** is a function that is false only when the antecedent is true and the consequent is false.
- **Material equivalence** is a function such that both sides always have the same truth-value.

There are several methods to prove the validity of an argument in propositional logic. The default method introduced here is the truth-table method. It operates on a single simple idea: leave nothing hidden by showing all possible truth-value assignments to propositions in an argument. If whenever all premises are true the conclusion is true, then the argument is valid. If there is one case in which the premises are all true yet the conclusion

is false, then the argument is invalid. The truth-table method is exhaustive, complete, and mechanically decidable. However, it can also be time-consuming and inconvenient.

The short truth-table method uses the same principle of the truth-table method except that it considers only the possibility of an argument being invalid. If no contradiction is found under the assumption of invalidity, then an argument is indeed invalid. On the other hand, if there is contradiction, then the argument is valid. The short truth-table method enjoys the same advantages of exhaustiveness and completeness as the truth-table method and is more efficient. However, it is not always effective because some propositions may have many possible truth-value assignments. In such cases, we would need to go back to constructing a full truth-table.

Finally, we introduced the method of deduction, also called natural deduction. This method does not aim to give an overview of everything in a single shot but investigates step by step from the known into the unknown. Natural deduction accepts a number of inference rules and transformation rules. The more rules that are accepted, the more easily a proof can be constructed. This book introduced ten rules in each category. Some books may be slightly different. It also advises three strategies to construct a deduction:

STRATEGIES OF NATURAL DEDUCTION

- **Backwards construction:** Start with the conclusion and think backwards on how it can relate to the premises given. Although we actually work backwards, we still present the deduction as deriving from the premises.
- **Conditional proof (CP):** If a conclusion is a conditional, we can prove the argument by assuming the antecedent of the conclusion and see if we can deduce the consequent. If so, then we discharge the additional assumption and prove the desired conclusion.

- ***Reductio ad absurdum*** or argument by contradiction (contra): We assume the negation of the conclusion and work out a contradiction. Then we can discharge the additional assumption by claiming that the negation of such negation is the case.

Different methods have different merits and limitations. The most important theoretical point, however, is that propositional logic is complete. This provides the justification of the truth-table method, which is the basis for everything else.

6

Predicate logic

In this chapter you will learn about:

- ***function, concept and quantification***
- ***translating categorical propositions into predicate logic***
- ***translating ordinary language***
- ***rules of inference and equivalence concerning quantification***
- ***deduction***

'Excellent!' I cried.

'Elementary,' said he. 'It is one of those instances where the reasoner can produce an effect which seems remarkable to his neighbour, because the latter has missed the one little point which is the basis of the deduction.'

Arthur Conan Doyle (1893), 'The Crooked Man', *The Memoirs of Sherlock Holmes*

6.1 Function, concept, quantification

We have introduced categorical logic and propositional logic in the last two chapters. Both systems are sound, complete, and highly applicable to ordinary reasoning. However, they also have their limitations. In this chapter we introduce a new logical system, predicate logic, to overcome these issues.

Propositional logic examines only the *external*, truth-functional relations among propositions. However, it is unable to determine the validity of arguments which depend on *internal* relations of components in a proposition. For example:

Example (1): All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.

Each premise and conclusion is an atomic proposition. Hence, each should be represented by a simple letter under propositional logic. Since each is a different proposition, different symbols should be assigned. Let p = all men are mortal, q = Socrates is a man, and r = Socrates is mortal. The argument is symbolized as:

$$\begin{array}{l} p \\ \underline{q} \\ \therefore r \end{array}$$

We can hardly see any connections between these propositions in this form. So, it is also impossible to determine the validity of this argument using propositional logic. However, we

know this is a valid argument, because the three propositions are connected by some common components within the propositions, such as humanity and mortality. A logic would be much more powerful if it could analyse internal structures of propositions as well.

Categorical logic analyses the *internal* structure of a proposition. However, it can only deal with simple predicates and a simple range of relations between subjects and predicates, specifically the relations of quality and quantity. The predicates to be dealt with in categorical logic are called one-place predicates, in contrast to multiple or *n*-place predicates. An **n-place predicate** predicates *n* number of parties. A one-place predicate concerns only one party; for example, 'is wise' is a one-place predicate to apply to a person such as Socrates, 'is white' is a one-place predicate to qualify snow or paper, etc. Most one-place predicates are adjectives and verbs. A two-place predicate, such as 'love', 'being a sibling of', 'north of', etc. connects two parties in a specific order, for instance, 'John loves Mary' is different from 'Mary loves John'. Two-place predicates are normally what we commonly call relations. Three-place predicates connect three parties or ascribe three aspects of things. For example, 'send' is a three-place predicate: someone *x* sends someone *y* something *z*. Another example: freedom, though usually considered one-place, is indeed a three-place predicate; freedom is individuated always by a three-part formula, 'freedom for *x* (an agent) to do *y* (take some action) from *z* (avoid the intervention of someone or something)'. Hence, a smoking ban enhances freedom for non-smokers to enjoy good health by being protected from second-hand smoke, while at the same time it curtails freedom for smokers to smoke wherever they want without lawful intervention of other people.

Categorical logic typically deals with one-place predicates only. Of course, it is also possible to transform an *n*-place predicate into one-place; for instance, instead of treating 'love' as a predicate, we can consider 'loving Mary' as the predicate. So, 'John loves Mary' becomes 'John is a person who loves Mary'. However, given that categorical logic is typically used

in syllogism which only entertains three terms, manipulating predicates like that is likely to run the risk of creating too many predicates for a syllogism. For example:

Example (2): John loves Mary.

Mary loves shopping.

Therefore, John loves shopping.

The argument becomes:

John is a person who loves Mary.

Mary is a person who loves shopping.

Therefore, John is a person who loves shopping.

The argument so formulated contains four terms rather than three. Let S = John, P = a person who loves shopping, M = Mary, then a fourth term N has to be created to stand for persons who love Mary. Mary and a person who loves Mary are different sets of objects. So the form of the argument is then:

S is N

$\underline{\text{M is P}}$

∴ S is P

There is no middle term to connect the major and the minor term. Categorical syllogism cannot determine validity for this kind of argument.

Nor can categorical logic handle predicates with complex structures. For example:

Example (3): All wealthy people can join the club.

Some highly talented people are invited to join the club.

Therefore, all club members are either wealthy or highly talented.

Let M = club members, S = wealthy people, P = highly talented people.

All S is M

<u>Some P is M</u>

∴ All M is (S or P)

The conclusion is not one of the four categorical propositions (A, E, I, O). So, categorical logic cannot handle this kind of argument either.

We naturally wonder if there is any attempt to combine the two approaches and form a more powerful system that covers sentences of greater complexity and variety. Indeed, there is! Predicate logic does exactly that.

Predicate logic is sometimes called the quantification theory. Its most important contributor is the German logician Gottlob Frege (see Chapter 1), who is recognized as the father of modern logic and analytic philosophy. Some of his most important works on logic include *Begriffsschrift* (*Concept Script*, 1879) and *The Foundation of Arithmetic* (1884).

The basic insight of predicate logic is to take *predicate as a function*, but not an object. We have already seen (in Chapter 5) that in mathematics and logic, a function is an operation which maps a certain object to another. In the case of a predicate, it maps an object to truth or falsity. A predicate is like a machine with an empty space waiting for something to be input so that it will output something as programmed. For example, 'is white' should literally be understood as having this structure: '__ is white'. Suppose we input snow in the space before 'is'. Since snow indeed satisfies the property of being white, i.e. 'snow is white' is true, the output is Truth. In another case, suppose we input 'coal'. Coal is not white, so the combination of coal and the property of being white leads to Falsity instead.

Frege took Truth and Falsity (both capitalized) as objects; this may sound weird but let us grant him that. So snow, coal, Truth, Falsity are objects, yet being white is *not* an object. Objects are ontologically complete; they stand alone by themselves. However, being white is not complete by itself but has an empty space waiting to be filled. Frege called a function like this a concept. A concept is a function that maps objects to truth-values. To use his

example, an animal that falls under the concept of *being a horse* and satisfies it is an object: it is an object horse. However, *being a horse* is not an object but a concept. So, as Frege famously said, the concept HORSE is not a horse. Frege used the term 'concept' to stand for a property.

We have to be careful that for Frege, a concept is not a mental entity, but is an abstract logical function. Similarly, Frege also used the term 'thought' to represent a proposition, which he regarded as an abstract entity, but not a psychological entity. A thought can be entertained or apprehended by a mind, but it does not have to be so. Just like a book standing on a shelf, even when no one reads it, the ideas or thoughts contained inside are still there. An idea not entertained by any mind is still an idea. So it is not by virtue of being thought of by a certain person that a thought comes into existence. Frege is a Platonist about thoughts.

Back to logic, so how does this idea of a predicate as a concept work for reasoning? A predicate is to be satisfied with an object. So predicate logic contains two basic types of elements: symbols standing for concepts or predicates, and symbols standing for objects. **Predicates** are represented by capital letters *F*, *G*, *H*, etc. For objects, there are in fact two types: definite objects called **individual constants**, represented by small letters *a*, *b*, *c*... and indefinite objects called **variables**, also represented by small letters such as *x*, *y*, *z*.

Predicates always need to be satisfied; hence, we always write a predicate together with an object. For a one-place predicate, one object is needed to satisfy it. So we get two types of structures: *Fa* or *Fx*. *Fa* is read as '*a* is *F*'. *Fx* is read as '*x* is *F*'. When an individual constant is applied to a predicate, the predicate will map a truth-value (either True or False), like the cases of applying snow to 'is white' generates truth, and coal to 'is white' generates falsity. So *Fa* is a proposition and it represents a complete thought. *Fa* is also called a **singular proposition**, which is defined as a proposition asserting that a particular individual has a specified attribute. However, when a variable is applied to a predicate, since the object is indefinite, no judgement can be made about whether the

output is truth or falsity. So Fx is not a proposition. Instead, Fx is called a **propositional function**. A propositional function is an expression composed of a predicate and a variable; it has no truth-values and must be either bounded by a quantifier (universal or existential) or substituted with an individual constant to become truth-evaluable.

A propositional function can have a truth-value if it is quantified. Indeed, in predicate logic, a propositional function must be bounded by a quantifier to become a well-formed formula. For example, 'x is wise' has no truth-value because no definite object is specified. However, 'for some x, x is wise' (meaning: something is wise) has a truth-value: it is true because something in the world is wise. 'For all x, x is wise' (meaning: everything is wise), however, is false because there are fools and so not everyone or everything in the world is wise. So, in general, for one-place predicates, there are only three types of **well-formed formulas**. A well-formed formula is roughly the same as a proposition, i.e. something can be either true or false.

Fa

For all x, Fx

For some x, Fx

We invent some symbols called quantifiers to stand for 'for all' and 'for some'. $\forall$ is the **universal quantifier** to stand for 'for all'. $\exists$ is the **existential quantifier** to stand for 'for some' or 'there exists some'. The above three types of formulas become:

Fa

$\forall x Fx$

$\exists x Fx$

For example:

'Socrates is wise', symbolized as Ws

'All objects are wise', symbolized as $\forall x Wx$

'Some object is wise', symbolized as $\exists x Wx$

A two-place predicate is represented in predicate logic as *Fab* or *Fxy*, where *a*, *b* and *x*, *y* are in a specific order. For example, let *j* = John, *m* = Mary, *L* = love, 'John loves Mary' is represented as *Ljm*. Changing the order would mean a completely different claim. *Lmj* would represent 'Mary loves John', for instance. Other n-place predicates are represented in a similar manner as *Fa...n* or *Fx...z*.

We should specially mention a predicate: identity. **Identity** is a two-place predicate denoting a relation, often expressed in English as 'is identical to', 'is the very same thing as', or simply 'is', etc. In predicate logic, it is expressed by the symbol '=' connecting two individuals (constants or variables) on either side. For example, 'Cicero is Tullius' is expressed as '$c = t$', where c stands for Cicero and t stands for Tullius. A formula containing an identity relation, such as $a = b$, is considered an atomic formula just like *Fa*. It is understood that a (hidden) bracket always holds around an identity relation. Following this line of thought, a negation of identity ('a is not identical to b') is a negation of a proposition, expressed as 'it is not the case that a is identical to b', rather than a negation of an individual, which does not make sense. For example, where b stands for Batman and s stands for Superman, 'Batman is not Superman' is expressed as '$\neg\ b = s$', not '$b \neg = s$' (the latter is considered not well-formed, i.e. ungrammatical, in predicate logic). Identity is not a logical connective connecting propositions.

Finally, predicate logic retains the same five common truth-functional connectives as propositional logic: negation ($\neg$), conjunction (&), disjunction ($\vee$), material implication ($\rightarrow$), and material equivalence ($\leftrightarrow$).

A well-formed formula is either an atomic formula or a truth-functional combination of well-formed atomic formulas. Only well-formed formulas are truth-evaluable, i.e. can be either true or false.

The above exhausts the semantics and syntax of predicate logic. Let us summarize.

Key idea: Predicate logic

Semantics

Predicate logic has the following vocabularies:

- **Predicates:** *F, G, H...,* =
- **Individual constants:** *a, b, c...*
- **Variables:** *x, y, z...*
- **Quantifiers:** ∀ (universal), ∃ (existential)
- **Connectives:** –, &, v, →, ↔

Syntax

A well-formed formula is either an atomic formula or a truth-functional combination of well-formed atomic formulas. There are only three types of atomic formulas:

- *Fa* (or, *Fa...n* in case *F* is a *n*-place predicate)
- ∀*xFx*
- ∃*xFx* (or, *ϕx..ϕzFx...z* in case *F* is a *n*-place predicate and ***ϕ*** is either ∀ or ∃)

6.2 Translating categorical propositions into predicate logic

We can now reinterpret the logical forms of the four categorical propositions in the formats of predicate logic. The A-proposition 'All S is P' claims that whatever is S is P. 'Whatever is S' means any objects that satisfy the property of S. 'P' means any objects that satisfy the property of P. The whole proposition 'whatever is S is P' thus claims that for any object, if an object satisfies S, then that object also satisfies P. Since this applies to any object, the quantification is universal. We use ∀ to construct the claim. Within the scope, there is a material implication relation between anything that is S and anything that is P. It is a material implication, rather than a conjunction, because according to modern interpretation, universal propositions such as A- and E-propositions have no existential import. They do not assert that there is indeed S, but

rather that if something is S, then it would have the property of being P. Therefore, the A-proposition should be rewritten in quantificational terms as:

All S is P: $\forall x\ (Sx \rightarrow Px)$

Example (4):

Let H = being human, M = being mortal.

All humans are mortal: $\forall x(Hx \rightarrow Mx)$ (read as: For all x, if x is human, then x is mortal.)

A material implication is true when its antecedent is false. Thus, $\forall x(Sx \rightarrow Px)$ is true when $\neg \exists xSx$ is true. This is what we need: A- or E-propositions can be true, yet it does not commit us to a position that S exists.

Similarly, the E-proposition 'No S is P' claims that for any object, if it satisfies the property of being S, then it would not satisfy the property of being P. The scope is universal. There is also no existential import concerning objects satisfying S. So the logical form of the E-proposition is as follows.

No S is P: $\forall x(Sx \rightarrow \neg Px)$

Example (5):

Let H = being homeless, W = being wealthy.

No homeless are wealthy: $\forall x(Hx \rightarrow \neg Wx)$ (read as: For all x, if x is homeless, then x is not wealthy.)

The I-proposition 'Some S is P' claims that some object satisfies both the properties of being S and being P. It is a particular claim, so it does have existential import, namely, it asserts that the object having such properties exists. Conjunction is used instead of implication, implying that $\exists xSx$ must be true whether or not an object satisfying the property of being S also satisfies that of being P. The translation is thus:

Some S is P: $\exists x(Sx \,\&\, Px)$

Example (6):

Let T = being a teenager, O = being outspoken.

Some teenagers are outspoken: $\exists x(Tx \& Ox)$ (read as: There exists x such that x is a teenager and x is outspoken.)

Similarly, the O-proposition 'Some S is not P' is a particular claim that some object satisfying the property of being S but not that of being P.

Some S is not P: $\exists x(Sx \& \neg Px)$

Example (7):

Let P = being a politician, H = being honest.

Some politicians are not honest: $\exists x(Px \& \neg Hx)$ (read as: There exists x such that x is a politician and x is not honest.)

Categorical logic is completely made up of four categorical propositions. Now we have represented all categorical propositions in quantified forms, that means we can deal with all categorical syllogisms by predicate logic, too. In fact, we can do more than just categorical syllogisms.

Formulas in predicate logic can have as many structures inside a proposition and as many different connectives between them as possible. It can also handle arguments with many terms, and is not restricted to three. In these ways, predicate logic frees categorical logic from its limited application.

Predicate logic also absorbs propositional logic. All atomic propositions in propositional logic are now rewritten as either of the three well-formed formula types.

6.3 Translating ordinary language

Predicate logic is renowned for its wide application to ordinary language and its ability to accommodate very complicated predicates. Let us start with something simple and work up to more complex examples. We will also look at some translations of arguments, although we can assume that once we know how to translate sentences, we should already know how to translate arguments.

The basic skill, of course, is to identify the predicates and the individuals first. If variables are involved, then we also need to identify and assign a suitable quantifier to each of them.

Example (8): London is pretty.

'Pretty' is a predicate. London is an individual. Let P = being pretty, l = London. The proposition is translated as:

Pl

Example (9): Everything is perfect.

'Perfect' is a predicate. 'Everything' indicates variable individuals. A universal quantifier is appropriate because 'every' certainly covers all things concerned. Let P = being perfect.

$\forall xPx$

Example (10): Nothing is perfect.

(10) is similar to (9) in that it has the same predicate and it also involves variable individuals. A universal quantifier is suitable, too, because 'nothing' covers all things: it refers to all things and asserts of them all that they do not have a certain attribute, which is being perfect.

Let P = being perfect. (10) is translated as:

$\forall x\neg Px$

When nothing is perfect, it means it is not the case that something is perfect. Remember in categorical logic, the contradictory of an E-proposition is an I-proposition. So an E-proposition is equivalent to the negation of an I-proposition. The above formula can also be written as:

$\neg\exists xPx$

Indeed, in general, the following rules of equivalence or transformation rules hold for predicate logic:

For any predicate F:

i $\forall x\neg Fx \equiv \neg\exists xFx$

ii $\exists x\neg Fx \equiv \neg\forall xFx$

iii $\forall xFx \equiv \neg\exists x\neg Fx$

iv $\exists xFx \equiv \neg\forall x\neg Fx$

These rules reflect the ideas of contradictories in categorical logic. The first rule corresponds to the idea that an E-proposition is the negation of an I-proposition. The second rule: an O-proposition is the negation of an A-proposition. The third rule: an A-proposition is the negation of an O-proposition. The fourth rule: an I-proposition is the negation of an E-proposition.

Note that the negation of 'everything is perfect' is not 'nothing is perfect'. The negation of 'everything is perfect' is $\neg\forall xPx$, read as 'it is not the case that everything is perfect'. To fulfil this claim, it only requires finding one thing which is not perfect, i.e. $\exists x\neg Px$, read as 'something is not perfect'. It does not require that everything is not perfect, i.e.$\forall x\neg Px$, or that there is not anything that is perfect, i.e. $\neg\exists xPx$.

In general, we get a very important lesson: that is, the **scope of quantification** is a major determining factor for a formula. Changing the position of a symbol, or a quantifier, or adding a bracket, etc. would completely change it to a different assertion. Logic is very precise. The above are cases in point. To reiterate,

$$\forall x\neg Px \equiv \neg\exists xPx$$

$$\exists x\neg Px \equiv \neg\forall xPx$$

But:

$$\forall x\neg Px \not\equiv \neg\forall xPx$$

$$\exists x\neg Px \not\equiv \neg\exists xPx$$

Example (11): Everything is imperfect.

Suppose we take 'imperfect' as the opposite of the predicate 'being perfect'. 'Everything is imperfect' thus means the same as 'everything is not perfect'. (11) is the same as (10) 'Nothing is perfect'. The proposition can be translated as:

$$\forall x\neg Px$$

Or, equivalently:

$\neg\exists x Px$

Example (12): Something is beautiful.

Example (13): There is at least one thing that is beautiful.

Let B = being beautiful. 'Something is beautiful' is written as:

$\exists x Bx$

In logic, 'some' is always taken to mean 'at least one'. We have encountered that discussion in categorical logic already. It is the same in predicate logic. So, the translation of (13) is the same as that of (12), i.e. $\exists x Bx$. The case, however, is very different when we consider not 'at least one', but 'at most one' or 'exactly one'.

Example (14): There is exactly one thing that is beautiful.

'Exactly one' means there is at least one and there is at most one. 'There is at least one' means that there cannot be none, or zero object, that satisfies the required predicate. 'There is at most one' means it cannot be more than one either. It is more difficult to indicate this latter requirement. We have to imagine its opposite. 'There is at most one' means that it is not possible for two objects to satisfy our required predicate yet they are not the same object. This implies that, if there are two objects satisfying our required predicate, then those two objects must be identical. This makes it the case that there can only be one object that satisfies the predicate. We use such a situation to express what it is to assert that there is exactly one thing which is so and so.

Let B = being beautiful. Now we need two variables, rather than one to express the impossibility of identity. (14) is translated as follows.

$\exists x Bx \ \& \ \forall x \forall y[(Bx \ \& \ By) \rightarrow x = y]$

The first part of this proposition ($\exists x Bx$) reads: there is something x which satisfies the property of B. This fulfils the requirement that there is at least one thing which is beautiful. The existential quantifier is appropriate because the proposition does assert that there *is* something beautiful.

The second part of the proposition indicates the requirement that there is at most one beautiful thing. It does so by supposing that if there are two objects, any objects, which satisfy the requirement of being beautiful, then those two objects are identical. Since it is about any object and it is just a supposition, not asserting that there are indeed two such objects, universal quantification rather than existential quantification should be used. $\forall x \forall y[(Bx \,\&\, By) \rightarrow x = y]$ reads: For all x and all y, if x is beautiful and y is beautiful, then x is identical to y.

This is a complicated one, so take the time to reflect on it and make sure you understand it. Indeed, Bertrand Russell famously explicated definite descriptions as having a logical structure involving 'exactly one'. In his seminal paper 'On Denoting' (1905), he analysed sentences containing definite descriptions such as 'the present King of France' as making three claims:

'The present King of France is bald' is true if and only if:

i There is a present King of France,

ii There is exactly one present King of France, and

iii Whoever is the present King of France is bald.

In symbols, let F = being the present King of France, B = being bald, 'The present King of France is bald' is translated as:

$$\exists x[Fx \,\&\, \forall y(Fy \rightarrow x = y) \,\&\, Bx]$$

'There is a present King of France' is expressed by $\exists xFx$. 'There is exactly one present King of France' should be expressed as $\forall x \forall y[(Fx \,\&\, Fy) \rightarrow x = y]$. However, since it is already asserted in the first part that there is indeed an x which satisfies F, putting the second part within the quantification of the first part means we do not need to add a universal quantification again. It is not only the case that *if* there is an x that satisfies F and a y that satisfies F, then x is identical to y. It is given (by the first part of the claim) that there *is* indeed an x that satisfies F. So we only need to further quantify y, but not x. The third part of the claim, standing by itself, should be written as $\forall x(Fx \rightarrow Bx)$. However, the first part of the proposition already asserts some x is F, that means the antecedent of this conditional is true.

Or, equivalently:

$\neg\exists xPx$

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'There is a present King of France' is expressed by $\exists xFx$. 'There is exactly one present King of France' should be expressed as $\forall x \forall y[(Fx \,\&\, Fy) \rightarrow x = y]$. However, since it is already asserted in the first part that there is indeed an x which satisfies F, putting the second part within the quantification of the first part means we do not need to add a universal quantification again. It is not only the case that *if* there is an x that satisfies F and a y that satisfies F, then x is identical to y. It is given (by the first part of the claim) that there *is* indeed an x that satisfies F. So we only need to further quantify y, but not x. The third part of the claim, standing by itself, should be written as $\forall x(Fx \rightarrow Bx)$. However, the first part of the proposition already asserts some x is F, that means the antecedent of this conditional is true.

To make the conditional true, it *must* be the case that some x is B is true. We thus write Bx under the *existential* quantification of Fx and it is now a conjunction rather than a conditional.

Example (15): Something is not beautiful.

Example (16): Not everything is beautiful.

Example (17): It is not the case that nothing is beautiful.

Let B = being beautiful. (15) is easy; it should be written as $\exists x \neg Bx$. (16) is written as $\neg \forall x Bx$. When something is not beautiful, it implies that not everything is beautiful. So (15) and (16) are indeed equivalent. The rules of equivalences stated earlier already capture this.

(17) is a double negation. 'Nothing is beautiful' is translated as $\forall x \neg Bx$ or equivalently, $\neg \exists x Bx$. The latter is already a negation, meaning it is not the case that something is beautiful. To negate it again implies an affirmation. So (17) is translated as 'Something is beautiful', $\exists x Bx$.

$\neg\neg\exists x Bx$ is equivalent to $\exists x Bx$

Example (18): Peter is the cousin of Quinn.

Let us try some multi-place predicates. 'Being the cousin of' is a two-place predicate. Let Cxy = x is a cousin of y, p = Peter and q = Quinn. (18) is symbolized as:

Cpq

Note in this case, being a cousin is a symmetric relation. A relation R is a **symmetric relation** if an individual a is related to another individual b, then b is related back to a in the same way; in symbols, Rab implies Rba. If Peter is a cousin of Quinn, Quinn is also a cousin of Peter. However, not all relations are symmetric. Obviously, 'being the boss of' is not a symmetric relation. If Jack is the boss of Bob, then Bob is not the boss of Jack, in the same sense of being a boss.

There are actually three features to analyse in a relation. Apart from being symmetric, a relation can also be classified according to its being transitive and reflexive. A **transitive relation** R is one that can be carried forward; in symbols, Rab and Rbc implies

Rac. For example, 'larger then', 'older than' are transitive relations. The number 9 is larger than 7 and 7 is larger than 3, so 9 is larger than 3. A counter-example is love: if John loves Mary and Mary loves Peter, it certainly does not imply that John loves Peter. Indeed, probably on the contrary, John would hate Peter for Mary's loving Peter, rather than himself. Love is also not a symmetric relation: John loves Mary does not imply Mary loves John.

Finally, a **reflexive relation** is one that bestows upon itself; in symbols, if a relation R is reflexive and a is any member of a domain, then Raa always holds. Identity, for example is reflexive because any object is identical to itself. Equality ('is equal to') is also reflexive upon the set of all real numbers; e.g. '1 is equal to 1'. Love is arguably a reflexive relation because people naturally love themselves and try to protect their own interests. However, there can also be exceptions; some people may hate themselves and be self-destructive. So love is not a reflexive relation after all.

We need to be aware of features of a relation because sometimes they may affect what is claimed. Moreover, scope is again an important issue. Let us examine the following examples.

> Example (19): Everyone loves someone.
>
> Example (20): Someone is loved by everyone.
>
> Let Lxy = x loves y. Consider two possible candidates for the translation. Which translation below is the correct translation of (19)? Which is the correct translation of (20)?
>
> a $\forall x \exists y(Lxy)$
>
> b $\exists x \forall y(Lyx)$

(a) means that for all individuals x, there is another individual y, such that x loves y. Different individuals may love different people. This is what 'Everyone loves someone' means. So (a) is the correct translation of (19). However, (b) means that there is some individual x, such that for all individuals y, y loves x (or x is loved by y). Here, everyone loves the same person. It does not mean different individuals may love different people. It is the meaning of (20), rather than (19). Hence, although in English,

we are often taught that active and passive voices are inter-translatable, e.g. 'Andrew takes out the fire' means the same as 'the fire is taken out by Andrew', logically, they are not always the same. We need to figure out exactly what truth-conditions are involved case by case.

We describe the difference between (19) and (20) as they have different scopes. In (19), the universal quantifier quantifies x and the existential quantifier quantifies y, but in (20), the existential quantifier quantifies x and the universal quantifier quantifies y. Moreover, in (19), the universal quantifier takes the wide scope (located in the outermost position of the proposition), such that the other quantifier has to work under this quantification. Whereas in (20), the existential quantifier takes the wide scope and the universal quantifier works under the existential quantification.

Scope confusion may sometimes cause errors in reasoning. It may even cause some deep philosophical troubles. There is a famous argument for the existence of God called the causal argument, or the argument of the first cause. The argument runs roughly as follows: Because everything has a cause, eventually there must be a cause that causes everything. God in creating the world is that first cause. Effects exist, so causes must exist. Therefore, the first cause exists, and so God exists.

This argument depends on two crucial propositions, (21) and (22), and argues that (21) implies (22). Let us examine whether they are necessarily connected.

Example (21): Everything has a cause.

Example (22): There is a cause of everything.

Causation is a two-place predicate. Let $Cxy = x$ causes y. (21) is translated as (21'). However, (22) claims (22'), i.e. (22) is claiming something very different from (21).

(21') $\forall x \exists y Cyx$ (read as: For all x, there is some y such that y causes x.)

(22') $\exists y \forall x Cx$ (read as: There is some y, such that for all x, y causes x.)

(21') and (22') have completely different scopes of quantification. One can hardly deduce (22') from (21'). The causal argument for the existence of God is thus flawed. It is logically analogous to arguing from (23) to (24), which is obviously and intuitively known to be invalid. (23) is obviously true, but (24) false. This argument shares exactly the same logical structure as arguing (22) from (21). We can say so by comparing their logical structures (23') and (24') with (21') and (22') respectively.

Example (23): Everyone eats something.

Example (24): There is something everyone eats.

Let Exy = x eats y:

(23') $\forall x \exists y Eyx$

(24') $\exists y \forall x Eyz$

The method of arguing for the validity (or invalidity) of an argument by constructing valid (or invalid) arguments of the same logical structure is called the method of constructing a **logical analogy.**

Example (25): All trespassers will be expelled or prosecuted.

Example (26): All that glitters is not gold.

Example (27): None but the brave and the honest deserve the fair.

The next set of examples shows the use of logical connectives. (25) means that if one trespasses, then one is either expelled or prosecuted. It does not identify any particular person who would receive such treatment. Indeed, *anyone* who trespasses would be treated so. Hence, the proposition is (i) about variable individuals rather than particular constants; and (ii) the quantification should be universal rather than existential. Indeed, it does not assert that there are actually any trespassers at all. All trespassers would be treated a certain way, so it is an implication relation. The way to treat a trespasser is composed of a disjunction (i.e. expelled or prosecuted); they are both consequent of being a trespasser. So, a bracket for the consequent is necessary, too. All in all, (25) should be translated as follows.

Let T = be a trespasser, E = be expelled, P = be prosecuted

(25') $\forall x[Tx \rightarrow (Ex \vee Px)]$

(26) is straightforward.

Let G = glittering, D = being gold.

(26') $\forall x(Gx \rightarrow \neg Dx)$

For (27), again this is a universal statement, so the main operator is implication. The subject has several attributes; hence, a bracket is needed to contain all those attributes before applying implication. 'None but' means 'only'. Thus, (27) means 'only the brave and the honest deserve the fair'. This does not mean that if one is brave and honest, then one deserves the fair. Rather, it means if one deserves the fair, then one must be brave and honest. Just as in propositional logic, 'only p, then q' is translated as '$q \rightarrow p$', rather than '$p \rightarrow q$', we also need to be cautious that in predicate logic, a similar rule of translation applies.

Let B = being brave, H = being honest, F = deserves the fair. (27) is translated as:

(27') $\forall x[Dx \rightarrow (Bx \;\&\; Hx)]$

Let us try more examples with 'only'. Note how the following propositions differ in meaning.

Example (28): Only British citizens can vote in the Brexit referendum.

Example (29): British citizens can only vote in the Brexit referendum.

Let B = being British citizens, V = can vote in the Brexit referendum.

(28') $\forall x(Vx \rightarrow Bx)$ or equivalently, $\forall x(\neg Bx \rightarrow \neg Vx)$

(29) however means not only that being a British citizen entitles one to vote in the Brexit referendum, but also that being a British citizen only entitles one to vote in the Brexit referendum but not in any other elections. So the subject matter is not British citizens but the elections in which a British citizen can vote. We should use different predicates from (28).

Let E = being an election in which a British citizen can vote, R = being the Brexit referendum, the variable x then denotes an election, rather than a person.

(29') $\forall x[(Ex \to Rx) \,\&\, (Rx \to Ex)]$, which is equivalent to $\forall x(Ex \leftrightarrow Rx)$

Remember in propositional logic, apart from 'only', we also discussed 'unless'. Let us see how to handle 'unless' in predicate logic.

Example (30): All will suffer unless someone sacrifices.

Example (31): All will suffer unless someone sacrifices and everyone is grateful to those who sacrifice.

Let U = suffer, A = sacrifice, Gxy = x is grateful to y ('being grateful' is a two-place predicate). (30) only uses U and A. It can be translated as follows.

(30') $\forall x \exists y(Ay \vee Ux)$, or equivalently

(30") $\forall x \exists y(\neg Ay \to Ux)$, or equivalently,

(30"') $\forall x \exists y(\neg Ux \to Ay)$

The translation of 'unless' in predicate logic pretty much follows the same rule as that in propositional logic. In propositional logic, 'unless A, B' is translated as 'A ∨ B', or '¬A → B' or '¬B → A'. In predicate logic, we just need to also take care about the quantification. In this example, those who suffer are quantified universally ('all will suffer') though there is only someone who sacrifices such that an existential quantifier is appropriate for the latter. It is important to make the variable agree with the relevant predicate.

(31) has a more complex clause following 'unless'. We need to bracket the whole thing before dealing with the rest.

(31') $\forall x \exists y[(Ay \,\&\, Gxy) \vee Ux)$, or equivalently

(31") $\forall x \exists y[(\neg Ay \,\&\, Gxy) \to Ux]$, or equivalently,

(31"') $\forall x \exists y[\neg Ux \to (Ay \,\&\, Gxy)]$

Our last example (stated below) involves a complex structure: something has to be a donkey, then whoever owns that donkey beats the donkey. Owning and beating are both two-place predicates. And the same donkey has to be owned and beaten by the same owner. So, this proposition involves at least two variables, x and y. The subject of the proposition 'everyone' suggests that a universal quantifier should apply to the owner. Donkeys do not have to be owned. However, if they are owned, they will be beaten. So a universal quantifier should be used for donkeys, too.

Example (32): Everyone who owns a donkey beats it.

Let D = being a donkey, Oxy = x owns y, Bxy = x beats y.

(32') $\forall x \forall y\ [(Dy \,\&\, Oxy) \rightarrow Bxy]$

Once we know how to translate propositions, it follows that we also know how to translate arguments. Below are the translations of the arguments already mentioned above.

Example (1): All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.

Let H = being a man, M = being mortal, s = Socrates.

(1') $\forall x(Hx \rightarrow Mx)$

$\underline{Hs\qquad\qquad}$

$\therefore Ms$

Example (3): All wealthy people can join the club.

Some highly talented people are invited to join the club.

Therefore, all club members are either wealthy or highly talented.

Let W = being wealthy, C = being a club member, H = being highly talented

$$(3') \; \forall x(Wx \rightarrow Cx)$$

$$\underline{\exists x(Hx \,\&\, Cx)}$$

$$\therefore \forall x[Cx \rightarrow (Wx \vee Hx)]$$

Exercise 6.1: Writing ordinary sentences in predicate logic

Translate each of the following into the logical notation of predicate logic.

1 No boy scout ever cheats.
2 Diplomats are not always rich.
3 Spider bites are sometimes fatal.
4 Not every application is successful.
5 Only a qualified mechanic can repair a car.
6 Unless one can swim, one cannot dive or row a boat.
7 Anyone who loves detective stories admires Sherlock Holmes or one is a fool.
8 Betty will only love exactly one person at a time.
9 All soldiers obey their superiors. Therefore, a superior is obeyed by every soldier.
10 Every soldier who disobeys their superiors loses the chance of being praised by them.

6.4 Rules of inference and equivalence concerning quantification

We have already mentioned that predicate logic no longer adopts the Venn diagram method or the truth-table methods. This is because predicate logic is more complicated than categorical syllogism and it deals with quantification, too. Propositional functions such as Sx and Px are not truth-valuable; so, although we can assign truth-values to a formula such as $\exists x(Sx \,\&\, Px)$, we cannot assign any truth-values to its components. Moreover, there are usually many functions in a proposition or an argument in predicate logic; therefore, even if a truth-table can be constructed, it is usually very long and complex.

However, we retain the method of natural deduction in predicate logic, and all strategies of natural deduction introduced in Chapter 5 apply. We only need to add a few more rules to govern the operations of quantifiers. We shall only introduce very simple derivations in predicate logic here, so only four quantifier rules of inference are listed. More rules of inference will be needed to deal with more complex arguments, such as the elimination and introduction of identity. For now, the four quantifier rules are as follows:

QUANTIFIER RULES OF INFERENCE

- Universal Elimination (∀E, also called Universal Instantiation UI): $\forall xFx$ / ∴ Fa (where a is any arbitrarily selected individual constant)
- Universal Introduction (∀I, also called Universal Generalization UG): Fa / ∴$\forall xFx$ (where a is any arbitrarily selected individual constant)
- Existential Elimination (∃E, also called Existential Instantiation EI): $\exists xFx$ / ∴ Fa (where a is any individual constant not previously occurred in the context)
- Existential Introduction (∃I, also called Existential Generalization EG): Fa / ∴ $\exists xFx$ (where a is any arbitrarily selected individual constant)

TRANSFORMATION RULES (OR, RULES OF EQUIVALENCE)

- Quantifier exchange (QE):

  $\forall x \neg Fx \equiv \neg\exists xFx$

  $\exists x \neg Fx \equiv \neg\forall xFx$

  $\forall xFx \equiv \neg\exists x\neg Fx$

  $\exists xFx \equiv \neg\forall x\neg Fx$

We have already introduced the rules of equivalence in Section 6.3 when we discussed translation. These rules of equivalence are consistent with the idea that A- and O-propositions, and E- and I-propositions are contradictories in categorical logic, so they should not be controversial.

Let us explain more about the rules of inference. This set of rules is tasked with eliminating and introducing quantification. These steps are necessary because we need to deal with operations within the scope of a quantifier, yet propositional functions have no truth-values so they cannot be operated on their own. Therefore we need to get rid of the quantifiers momentarily by **instantiating** a quantification. That means, we construct a model for quantification so that, instead of using quantifiers and variables, we can now express formulas solely in terms of individual constants. Each component within the original quantification becomes an atomic proposition, not bound by a quantifier, and thus can be freely operated upon. To get back to the quantified result, we reintroduce quantification in the end of a proof by **generalizing** atomic propositions generated in the process. We will look at how this works through some examples in Section 6.5.

Why should instantiation and generalization work in the way stated by these rules? Predicate logic assumes that all terms denote, including individual constants, variables and predicates. That means, in predicate logic although x is a variable, the fact that x is used in an inference implies that some x is assumed to exist. It is just that x is not any specific individual. Hence, from the premise that all x satisfies the function F, it must be the case that in a given domain, a particular individual, call it a, arbitrarily selected, must satisfy the function F. Universal instantiation is therefore justified.

Likewise, if there is an arbitrarily selected individual a which satisfies the function F, then we can generalize that all x satisfies F. Let us imagine the following case. Suppose a is an arbitrarily selected triangle and it is proven that the sum of its three internal angles is 180°. Then we know it is not only this particular triangle which has the sum of its internal angles being 180°. Rather, all triangles have the same property. It is because a is just an arbitrary triangle and the laws of geometry are universal. If we prove a case on one triangle, then we prove the case on all triangles. So we can generalize. Using the same reasoning, we can therefore accept the universal generalization rule.

You may question that in scientific investigations we do sometimes find cases that cannot be generalized. This is true

because we may indeed have false assumptions; for instance, we falsely postulate some patterns as universal. Yet we can accept this possibility of fallibility and continue to work until a contradiction is found to reject the assumptions. It is still more reasonable to accept the universal generalization rule than not applying any generalization at all.

The claim that all terms denote is not to be confused with the issue about existential import. In categorical logic, universal propositions (A- or E-) have no existential import, meaning that even when they are true, there is no implication that some object satisfying the property of *S* exists (i.e. $\exists xSx$ is true). However, this is a separate issue from claiming whether any object (x) exists at all. Predicate logic assumes that variables are objects existing in the domain, even though they are indeterminate. The idea that universal propositions have no existential import is reflected in predicate logic in the choice of the operation used to express the relations between the two predicates *S* and *P*. Material implication is chosen for universal propositions, but not conjunction. Likewise, the idea that particular propositions have existential import is also reflected in the choice of the operation to connect *S* and *P*. Conjunction is used instead of implication, suggesting that $\exists xSx$ must be true if $\exists x(Sx \,\&\, Px)$ or $\exists x(Sx \,\&\, \neg Px)$ is true.

It is easy to justify existential instantiation and generalization since no universality is claimed. Existential instantiation says that if something satisfies a function *F*, then some (particular) thing must satisfy that function. Existential generalization claims that if a particular object satisfies *F*, then there must be some object that satisfies *F*. Both claims are obvious and self-evident.

However, there is a restriction on what *a* is when we apply existential instantiation. In $\exists$E, *a* is not any arbitrarily selected individual but has to be something that does not already occur in the context of the argument concerned. This restriction is necessary and understandable. If no such restriction applies, then it is possible to instantiate one existential formula to an individual constant and then instantiate another existential formula to the same constant, when in fact the same individual

may not have properties from both formulas. Even if the individual has properties ascribed by both formulas, it may be purely accidental. So in general, it will be wrong to apply existential instantiation without any restriction.

This restriction implies that we should always apply ∃E before applying ∀E, ∀I or ∃I. Once we apply ∃E and assign a certain constant, we can still apply ∀E, for example, and use the same constant because there is no limitation on ∀E.

6.5 Deduction

Let us see how all the rules of inference and equivalences are put to work in the deduction of predicate logic.

Example (33): $\forall x(Fx \rightarrow Gx), Fa \,/\, \therefore\, Ga$

We cannot directly apply *modus ponens* in this case because the first premise is a quantified statement. We need to instantiate it first, so we can operate on its components separately. The whole bracket ($Fx \rightarrow Gx$) is under the universal quantifier ∀ and there is only one variable x, so when we instantiate the first premise, we can apply the same individual constant to the whole complex.

1	$\forall x(Fx \rightarrow Gx)$	
2	$Fa \,/\, \therefore\, Ga$	
3	$Fa \rightarrow Ga$	1 ∀E
4	Ga	3, 2 MP

By applying ∃E, we get $Fa \rightarrow Ga$. Now we can apply MP. We do have Fa from premise 2. Hence, we get the conclusion Ga.

Example (34): $\forall x(Fx \rightarrow Gx), \forall x(Gx \rightarrow Hx) \,/\, \therefore\, \forall x(Fx \rightarrow Hx)$

Intuitively we know that (34) is valid – it is hypothetical syllogism in quantified form! But we cannot directly apply the rule of HS because the propositions are quantified. So instantiation is necessary. At the end, after we get the desired result from applying HS, we need to generalize the result back again to get the specified conclusion.

1 $\forall x(Fx \rightarrow Gx)$

2 $\forall x(Gx \rightarrow Hx)$ / $\therefore \forall x(Fx \rightarrow Hx)$

3	$Fa \rightarrow Ga$	1 $\forall$E
4	$Ga \rightarrow Ha$	2 $\forall$E
5	$Fa \rightarrow Ha$	3, 4 HS
6	$\forall x(Fx \rightarrow Hx)$	5 $\forall$I

Example (35): $\forall x(Fx \rightarrow Gx)$, $\exists x(Fx \; \& \; \neg Hx)$ / $\therefore \exists x(Gx \; \& \; \neg Hx)$

This example if written in the form of categorical logic reads: All F is G, some F is not H, therefore some G is not H. We know from applying the Venn diagram method that it is a valid argument. Let us demonstrate now how we can deduce the argument in quantified forms.

1 $\forall x(Fx \rightarrow Gx)$

2 $\exists x(Fx \; \& \; \neg Hx)$ / $\therefore \exists x(Gx \; \& \; \neg Hx)$

3	$Fa \; \& \; \neg Ha$	2 $\neg\forall$E
4	Fa	3 Simp
5	$\neg Ha$	3 Simp
6	$Fa \rightarrow Ga$	1 $\forall$E
7	Ga	6, 4 MP
8	$Ga \; \& \neg Ha$	7, 6 Conj
9	$\exists x(Gx \; \& \neg Hx)$	8 $\exists$I

The following example shows how predicate logic can deal with more than three functions. This makes predicate logic more powerful than categorical logic, as categorical syllogism can only deal with three terms (i.e. three functions). Predicate logic, like propositional logic, can deal with as many terms and operations as are possibly made.

Example (36): $\forall x[Fx \rightarrow \neg (Hx \rightarrow Gx)]$, $\exists x \; (Hx \; \& \; Kx)$, $\forall x Fx$ / $\therefore \exists x(\neg Gx \; \& \; Kx)$

1	$\forall x[Fx \rightarrow \neg(Hx \rightarrow Gx)]$	
2	$\exists x(Hx \,\&\, Kx)$	
3	$\forall xFx \;/\therefore\; \exists x(\neg Gx \,\&\, Kx)$	
4	$Ha \,\&\, Ka$	2 ∃E
5	Ha	4 Simp
6	Ka	4 Simp
7	$Fa \rightarrow \neg(Ha \rightarrow Ga)$	1 ∀E
8	Fa	3 ∀E
9	$\neg(Ha \rightarrow Ga)$	7, 8 MP
10	$\neg(\neg Ha \vee Ga)$	9 Impl
11	$\neg\neg Ha \,\&\, \neg\, Ga$	10 DeM
12	$\neg Ga$	11 Simp
13	$\neg Ga \,\&\, Ka$	12, 6 Conj
14	$\exists x(\neg Gx \,\&\, Kx)$	13 ∃I

Like (35), we also need to do ∃E first in (36). Since there is only one existential premise, we do not have to do more ∃Es. The other ∀Es can also use the same individual constant as in the ∃E.

> Example (37): $\forall x(Fx \rightarrow Gx), \forall x(Lx \rightarrow Tx) /\therefore \forall x\,[\,(Fx \vee Lx) \rightarrow (Gx \vee Tx)]$

1	$\forall x(Fx \rightarrow Gx)$	
2	$\forall x(Lx \rightarrow Tx) \;/\therefore\; \forall x[(Fx \vee Lx) \rightarrow (Gx \vee Tx)]$	
3	$Fa \rightarrow Ga$	1 ∀E
4	$La \rightarrow Ta$	2 ∀E
5	$\neg Fa \vee Ga$	3 Impl
6	$(\neg Fa \vee Ga) \vee Ta$	5 Add
7	$\neg Fa \vee (Ga \vee Ta)$	6 Com
8	$(Ga \vee Ta) \vee \neg\, Fa$	7 Com

9	$\neg La \vee Ta$	4 Impl
10	$(\neg La \vee Ta) \vee Ga$	9 Add
11	$\neg La \vee (Ta \vee Ga)$	10 Com
12	$\neg La \vee (Ga \vee Ta)$	11 Com
13	$(Ga \vee Ta) \vee \neg La$	12 Com
14	$[(Ga \vee Ta) \vee \neg Fa] \,\&\, [(Ga \vee Ta) \vee \neg La]$	8, 13 Conj
15	$(Ga \vee Ta) \vee (\neg Fa \,\&\, \neg La)$	14 Dist
16	$(Ga \vee Ta) \vee \neg (Fa \vee La)$	15 DeM
17	$\neg (Fa \vee La) \vee (Ga \vee Ta)$	16 Com
18	$(Fa \vee La) \rightarrow (Ga \vee Ta)$	17 Impl
19	$\forall x[(Fx \vee Lx) \rightarrow (Gx \vee Tx)]$	18 ∀I

The point of (37) is that sometimes the conclusion is more complex than the premises. In any case, we try all means to drive the deduction to the form the conclusion requires. Sometimes addition and repositioning are necessary and we make use of all given rules to achieve the aim.

> Example (38): $\exists x(Fx \,\&\, Gx)$, $\exists x(Gx \,\&\, \neg Hx)$ / $\therefore \exists x(Fx \,\&\, \neg Hx)$

This is the equivalent of the following argument in categorical logic: Some F is G, Some F is not H, therefore, Some F is not H. We know that it is invalid. However, if we do not observe the limiting condition of ∃E, we might mistakenly prove it to be valid. Here is how.

1	$\exists x(Fx \,\&\, Gx)$	
2	$\exists x(Gx \,\&\, \neg Hx)$/ $\therefore \exists x(Fx \,\&\, \neg Hx)$	
3	$Fa \,\&\, Ga$	1 ∃E
4	$Ga \,\&\, \neg Ha$	2 ∃E (wrong!)
5	Fa	3 Simp
6	$\neg Ha$	4 Simp

7 $Fa \& \neg Ha$ 5, 6 Conj

8 $\exists x(Fx \& \neg Hx)$ 7 ∃I

This is a wrong proof because in step 4, we should not assign the same individual constant as in step 3. Premise 1 asserts something to have both properties of F and G, while premise 2 only asserts something to have both G and $\neg H$. Premise 2 does not assert that the same individual in 1 also has the properties of G and $\neg H$. Hence, we cannot assign the same individual constant a in both instantiations. If we do so, we are asserting that the same individual instantiates premise 1 and premise 2, which is not given from the premises. The correct way to do this is to assign different constants. So step 4 should be:

1 $\exists x(Fx \& Gx)$

2 $\exists x(Gx \& \neg Hx) / \therefore \exists x(Fx \& \neg Hx)$

3 $Fa \& Ga$ 1 ∃E

4 $Gb \& \neg Hb$ 2 ∃E

5 Fa 3 Simp

6 $\neg Hb$ 4 Simp

7 $Fa \& \neg Hb$ 5, 6 Conj

8 $\exists x(Fx \& \neg Hx)$ 7 ∃I (wrong!)

Once this is done, we see that we cannot proceed to step 8 and use existential generalization to convert the formula in 7 to a generalized form. It is because 8 requires that only one variable is bound by the existential quantifier and it is the same which satisfies F and $\neg H$.

This bring us to the topic of how to prove *invalidity* (rather than validity) of an argument in predicate logic. In principle, we can prove an argument to be invalid in quantified forms, too, using the short truth-table method. First we instantiate the argument and then we assume the argument to be invalid and work out a consistent truth-value assignment to all atomic propositions involved. If no contradiction is found, then the argument is invalid because one instance of invalidity is enough

to show that the argument in general is invalid. However, if we do find a contradiction in an instantiation, we do not thereby finish our job. The premises may generate the conclusion only in some particular instances but there is no guarantee that they always do. Hence, the general, quantified argument may still be invalid. So we must continue to test the argument with more instantiation until we find a case in which no contradiction is present.

In searching for instances, we observe two rules:

> Suppose there are exactly n individuals in the domain, $a, b, c \ldots n$, then
>
> $\forall xFx \equiv Fa \,\&\, Fb \,\&\, Fc \,\&\, \ldots Fn$
>
> $\exists xFx \equiv Fa \lor Fb \lor Fc \lor \ldots Fn$

This means that instances in universal quantification are connected by conjunction, while instances in existential quantification are connected by disjunction. If one instance is not enough to disprove an argument, we need to decide whether in the second instance the instances *Fa*, *Fb* or even *Fc* are conjuncts or disjuncts by observing the given relevant quantifier.

Let us use (38) as an example. If we only apply one instance to it, i.e. all x to be replaced by one individual constant a, the argument is valid. After instantiation, the argument becomes:

> $Fa \,\&\, Ga, Ga \,\&\, \neg Ha \,/ \therefore Fa \,\&\, \neg Ha$

$Fa \,\&\, Ga$	$Ga \,\&\, \neg Ha$	$Fa \,\&\, \neg Ha$
T T T	T T T F	T F F T

Assuming the instantiated argument is invalid, that is, all premises are true and the conclusion is false, we work out a contradiction: *Ha* has to be both true and false. So this means, when there is one and only one instance in the domain, the general argument in (38) is valid. This does not mean, however, that the general argument in (38) is valid, because there could be more objects available in the domain. We thus need to continue proving that (38) is invalid until we find a case where no contradiction is formed.

So, we try instantiation to two elements a and b. If this shows invalidity, then we have proved that the general argument is invalid. If not, then we need to continue instantiating three elements a, b, c, and so on until we find the invalid instance.

The general argument involves existential statements, so disjunction is used in instantiating more than one element.

$$(Fa \,\&\, Ga) \lor (Fb \,\&\, Gb), (Ga \,\&\, \neg Ha) \lor (Gb \,\&\, \neg Hb) \,/\, \therefore \\ (Fa \,\&\, \neg Ha) \lor (Fb \,\&\, \neg Hb)$$

$(Fa \,\&\, Ga) \lor (Fb \,\&\, Gb)$	$(Ga \,\&\, \neg Ha) \lor (Gb \,\&\, \neg Hb)$	$(Fa \,\&\, \neg Ha) \lor (Fb \,\&\, \neg Hb)$
T F FT T T T	F F FT T TT FT	T FFT F T F FT

Assuming the instantiated argument is invalid, then all premises are true and the conclusion is false. Since the conclusion is false and it is a disjunction, both disjuncts have to be false. However, there are still many possible assignments to make the two disjuncts false. Suppose we take *Fa* to be true, that given the conjunction is false, ¬*Ha* has to be false. Suppose *Fb* is true, the conjunction is false, so ¬*Hb* is false too. Apply these results to the premises. We can accommodate all of them and assign *Ga* and *Gb* consistent truth-values. No contradiction is found. That means the argument is invalid at least when *Fa* is T, *Ga* is F, *Ha* is T, while *Fb* is T, *Gb* is T and *Hb* is T. Any argument that can be invalid is invalid. And since the instantiated argument is an instance of the general argument of (38), the general argument of (38) is invalid too.

To sum up, we have seen that predicate logic uses the same basic skills of proving validity and invalidity as propositional logic. The difference is quantification, so some rules of instantiation and generalization are needed to enable transformation between quantified and non-quantified propositions. Existential instantiation (∃E) has a limitation and must be done before ∀E, ∀I and ∃I. This also implies that an argument containing two existential premises but one variable in its conclusion is rarely valid. This is because the existential premises would be instantiated into different individuals making it quite impossible to generalize back into something that contains only one variable.

It is harder to prove invalidity in predicate logic. Finding a contradiction in one instance is not good enough to prove that a quantified argument is not invalid. On the contrary, we need to test for invalidity until an instance with no contradiction is found. Universal quantification is translated as a conjunction of all component individual constants, while existential quantification is a disjunction.

Predicate logic is a powerful system, so there is a lot more content and techniques to explore. For example, there are rules to deal with more complex arguments, such as rules of identity elimination (=E) and introduction (=I). Scope is important in quantification. When more than one variable is present, we need to add scope lines to help manage and keep track of scopes during instantiation and generalization. This is a more advanced technique. Moreover, there are other methods of proof in predicate logic, such as the tree method (also called the tableaux method), which is very intuitive and becoming increasingly popular. Interested readers may delve deeper and discover more!

Exercise 6.2: Testing some simple inferences

Construct a formal proof of validity, or prove it invalid, for each of the following arguments. Symbolize the arguments wherever necessary.

1. $\forall x(Sx \rightarrow \neg Tx)$, $\exists x(Sx \;\&\; Ux)$ / $\therefore$ $\exists x(Ux \;\&\; \neg Tx)$
2. $\exists x(Px \;\&\; \neg Qx)$, $\forall x(Px \rightarrow Rx)$ / $\therefore \exists x(Rx \;\&\; \neg Qx)$
3. $\forall x(Gx \rightarrow Fx)$, $\forall x(Qx \rightarrow \neg Fx)$ / $\therefore \forall x(Qx \rightarrow \neg Gx)$
4. $\exists x\{(Ex \;\&\; Fx) \;\&\; [(Ex \vee Fx) \rightarrow (Gx \;\&\; Hx)]\}$ / $\therefore$ $\forall x(Ex \rightarrow Hx)$
5. $\exists x[(Cx \;\&\; \neg(Dx \rightarrow Ex)]$, $\forall x[(Cx \;\&\; Dx) \rightarrow Fx]$, $\forall x(Gx \rightarrow \neg Cx)$ / $\therefore \exists x(\neg Gx \;\&\; Fx)$
6. Everyone who makes the effort receives appropriate rewards. Andrew does not make the effort. Therefore, Andrew does not receive appropriate rewards.
7. Some managers are hardworking. Some officers are not hardworking. Therefore, no officers are managers.
8. All vintage cars are rare. Therefore, if someone owns a vintage car, someone owns something rare.

6.6 Chapter summary

The main point about predicate logic is that it creates fine structures within a proposition using function and object. It liberates the use of objects from concrete particulars already endowed with properties to indefinite variables. For the first time, logic can deal with quantification on a free scale. This system of logic is much more elegant and broad, making it more powerful than categorical logic or propositional logic on their own.

Predicate logic handles all inferences handled by categorical logic; we locate some general patterns to translate categorical propositions into quantified propositions. There are also some useful rules of equivalence, or transformation rules. Translating ordinary language into predicate logic is a more complicated matter. Several factors need to be taken into consideration and decided, including the choice of predicates, individual constants, variables, quantifiers and the scope of the quantifiers, etc. Note in particular that different scopes of quantification may mean completely different claims of the proposition.

TRANSLATION

- All S is P: $\forall x(Sx \rightarrow Px)$
- No S is P: $\forall x(Sx \rightarrow \neg Px)$
- Some S is P: $\exists x(Sx \,\&\, Px)$
- Some S is not P: $\exists x(Sx \,\&\, \neg Px)$
- $\forall x \neg Fx \equiv \neg \exists x Fx$
- $\exists x \neg Fx \equiv \neg \forall x Fx$
- $\forall x Fx \equiv \neg \exists \mathrm{x} \neg Fx$
- $\exists x Fx \equiv \neg \forall x \neg Fx$

In translation, we decide

1 what predicates are involved and the number of place(s) each predicate has

2 how many individual constants or variables are needed

3 the quantifier for each variable

4 the scope of each quantification.

Validity is proved in predicate logic by natural deduction. All rules applied to the natural deduction of propositional logic apply to predicate logic. Some further rules are added to govern quantification.

NATURAL DEDUCTION WITH RULES FOR QUANTIFICATION

The total list:

▶ Rules of inference

1 *Modus ponens* (MP): $p \rightarrow q, p$ / $\therefore q$

2 *Modus tollens (MT)*: $p \rightarrow q, \neg q$ / $\therefore \neg p$

3 Hypothetical syllogism (HS): $p \rightarrow q, q \rightarrow r$ / $\therefore p \rightarrow r$

4 Disjunctive syllogism (DS): $p \vee q, \neg p$ / $\therefore q$

5 Constructive dilemma (CD): $p \vee q, (p \rightarrow r)$ & $(q \rightarrow s)$ / $\therefore r \vee s$

6 Destructive dilemma (DD): $p \rightarrow r, q \rightarrow s, \neg r \vee \neg s, \therefore \neg p \vee \neg q$

7 Simplification (Simp): p & q / $\therefore p$

8 Conjunction (Conj): p, q / $\therefore p$ & q

9 Addition (Add): p / $\therefore p \vee q$

10 Absorption (Abs): $p \rightarrow q$ / $\therefore p \rightarrow (p$ & $q)$

11 Universal Elimination (∀E): $\forall x Fx$ / $\therefore Fa$ (where a is any arbitrary individual constant)

12 Universal Introduction (∀I): Fa / $\therefore \forall x Fx$ (where a is any arbitrary individual constant)

13 Existential Elimination (∃E): $\exists x Fx$ / ∴ Fa (where a is any individual constant not previously occurring in the context)

14 Existential Introduction (∃I): Fa / ∴ $\exists x Fx$ (where a is any arbitrary individual constant)

▸ Replacement (or, transformation) rules

1 Double negation (DN): $p \equiv \neg\neg p$

2 Commutation (Com): $(p \vee q) \equiv (q \vee p), (p \,\&\, q) \equiv (q \,\&\, p)$

3 Association (Assoc): $[p \vee (q \vee r)] \equiv [(p \vee q) \vee r], [p \,\&\, (q \,\&\, r)] \equiv [(p \,\&\, q) \,\&\, r]$

4 Distribution (Dist): $[p\& (q \vee r)] \equiv [(p \,\&\, q) \vee (p \,\&\, r)], [p \vee (q \,\&\, r)] \equiv [(p \vee q) \,\&\, (p \vee r)]$

5 De Morgan's laws (DeM): $\neg (p \,\&\, q) \equiv (\neg p \vee \neg q), \neg (p \vee q) \equiv (\neg p \,\&\, \neg q)]$

6 Transposition (Trans): $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$

7 Material implication (Impl): $(p \rightarrow q) \equiv (\neg p \vee q)$

8 Material equivalence (Equiv): $(p \equiv q) \equiv [(p \rightarrow q) \,\&\, (q \rightarrow p)], (p \equiv q) \equiv [(p \,\&\, q) \vee (\neg p \,\&\, \neg q)]$

9 Exportation (Exp): $[(p \,\&\, q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$

10 Tautology (Taut): $p \equiv (p \vee p), p \equiv (p \,\&\, p)$

11 Quantifier exchange (QE): $\forall x \neg Fx \equiv \neg\exists x Fx$, $\exists x \neg Fx \equiv \neg\forall x Fx$, $\forall x Fx \equiv \neg\exists x \neg Fx$, $\exists x Fx \equiv \neg\forall x \neg Fx$

To prove invalidity, we use the short-truth table method after instantiations.

▸ Rules of instantiation

Suppose there are exactly n individuals in the domain, $a, b, c \ldots n$, then

- $\forall x Fx \equiv Fa \,\&\, Fb \,\&\, Fc \,\&\, \ldots Fn$
- $\exists x Fx \equiv Fa \vee Fb \vee Fc \vee \ldots Fn$

Epilogue

Categorical logic and propositional logic are the most elementary logical systems. A typical university course in critical reasoning or elementary logic may explore categorical logic and propositional logic without going into predicate logic, because they are more relevant to daily life and are easier to handle. However, predicate logic is increasingly recognized as the standard within academic circles. It is so technically significant and powerful that the term 'classical logic' nowadays usually refers to this logic system. All college students majoring in philosophy should learn and master the basics of predicate logic because it is useful for philosophical analyses, especially in areas such as metaphysics, epistemology and philosophy of language.

Logic is a vast area. So far this book has only introduced a handful of classical logic systems. Classical logic respects three laws of logic: the law of self-identity ($a = a$, where a is an individual), the law of contradiction ($A \& \neg A$ is always false, where A is a proposition), and the law of excluded middle ($A \vee \neg A$ is always true). Some non-classical logics do not accept one or more laws of logic. Some do not accept the assumption that all terms denote (they are called free logic). Some other logics deal with time and temporal changes (temporal logic). Some deal with knowledge (deontic logic), provability (intuitionist logic), matters of degree (fuzzy logic). Some explore the area of possibilities (modal logic).

Moreover, there are also many logic paradoxes that puzzle and challenge everyone to explore beyond the usual boundaries of thought. Some interesting ones include Zeno's paradoxes (can a hare ever outrun a tortoise after giving it a head start?), the sorites paradox (where to draw the line between a heap and not a heap?), and the liar paradox (am I telling a lie if I say, 'I am a liar'?)

For further reading see 'Taking it further' at the back of the book. I hope this book has not only given you a good grasp of

the fundamentals of logic and critical reasoning, but also excited your interest in exploring the vast horizon of logic. Should that be the case, I would very much like to hear from you and share your joy! Find me on the internet, or email me at: sflee@hkbu.edu.hk.

Notes

[1] More on this research is 'The Failure of the Animal Model' (8 December 2015) and can be found online at: http://www.navs.org/what-we-do/keep-you-informed/science-corner/the-failure-of-the-animal-model/ (accessed on 4 March 2016)

[2] For an example, see https://www.youtube.com/watch?v=ey5D0kQWRLE (accessed 4 March 2016), or https://www.youtube.com/watch?v=MNJH1fr_z-g (accessed 4 March 2016)

[3] See the advert at: https://www.youtube.com/watch?v=ZzYldW0i7lI (accessed 4 March 2016)

[4] An example can be found here: https://www.youtube.com/watch?v=AxVnNIY4ppk (accessed 10 March 2016).

[5] In his national address on 19 March 2003, President George W. Bush announced the beginning of Operation Iraqi Freedom. He said: 'The people of the United States and our friends and allies will not live at the mercy of an outlaw regime that threatens the peace with weapons of mass murder.' However, many pre-war statements about Iraqi WMDs were not supported by the underlying intelligence.

[6] For the nature of causation: David Hume (1738, 1748), J L Mackie (1965), David Lewis (1973, 1979). For more fallacies related to causation, Moore and Parker (2009), Chapter 11.

[7] Different authors may present the rules in different ways; consequently, the number of rules also varies. For example, Copi and Cohen (2010) state six rules whereas Moore and Parker (2009) state three. I removed Copi and Cohen's rule on using exactly three terms because it is a matter of standard formulation rather than a rule of inference. Moore and Parker fused two rules about illicit major and illicit minor into one, forming Rule 3 here. But they omitted the rule on existential fallacy. In fact, their book does not mention the issue of existential import at all, which I consider a significant omission in conceptual issue.

Glossary

A posteriori That which requires empirical justification.

A priori That which does not depend on empirical evidence.

Abusive *ad hominem* When the argument is directly against persons, seeking to defame or discredit them. It is also called genetic fallacy, because it attacks at the genesis of an idea rather than the truth of its content.

Accent A shift of meaning arising as a consequence of changes in the emphasis given to its words or parts, thus leading to fallacious reasoning.

Accident An informal fallacy when one applies a generalization to an individual case that it does not properly govern.

Ambiguity An expression is ambiguous if it has more than one meaning.

Amphiboly An informal fallacy when one of the statements in an argument has more than one plausible meaning because of the loose or awkward way in which the words in that statement are combined.

Analytic statement Statement which is true or false by virtue of the meaning of the statement alone.

Appeal to common practice When an argument appeals to the fact that a view is universally or commonly held as support to its truth.

Appeal to emotion When careful reasoning is replaced with devices to create enthusiasm and emotional support for the conclusion advanced.

Appeal to force When careful reasoning is replaced with direct or insinuated threats to force the acceptance of the conclusion.

Appeal to ignorance A fallacy in which a proposition is argued to be true simply on the basis that it has not been proved false, or false simply because it has not been proved true.

Appeal to inappropriate authority When an argument appeals to a party having no legitimate claim to authority in the matter at hand.

Appeal to pity When an argument is not based on proper reasons but some unfortunate circumstance.

Appeal to populace Arguments sharing the form – 'Most people approve of X, therefore X is true.'

Apple polishing The strategy of praising someone in order to convince them to do the thing that you want.

Argument Structure that comprises a conclusion and some premises.

Atomic proposition The basic unit that expresses a complete thought.

Availability bias When we tend to retain and recall only information readily available to us recently or boosted via media coverage.

Axiom of double negation $\neg\neg p \equiv p$

Begging the question An argument begs the question when the reasoner assumes in the premises the truth of what he or she seeks to establish in the conclusion.

Bivalence The principle that there are only two possible truth-values for a proposition. Classical logic, including categorical logic, propositional logic and predicate logic, accepts bivalence.

Categorical syllogism A syllogism (an argument with exactly two premises and one conclusion) that comprises only categorical propositions.

Circular argument An argument in which the premises do not only support, but are in turn supported by the conclusion.

Circumstantial *ad hominem* When the attack is *indirectly* against persons, suggesting that they adopt their views chiefly because of their special circumstances or interests.

Cogent argument An argument is cogent if its premises are acceptable, relevant to and sufficient for its conclusion.

Completeness (of a logical system) A logical system is complete if every logical implication in that system is provable. In other words, for every truth-functionally valid argument, there is a way in this system to deduce the conclusion of that argument from its premises.

Complex proposition A proposition comprising several atomic propositions.

Complex question The argument assumes some unstated premises such that it misleads the audience into believing something which the responder cannot clarify by giving a simple answer.

Composition The fallacy of composition is made when (a) one reasons mistakenly from the attributes of a part to the attributes of the whole, or (b) one reasons mistakenly from the attributes of an individual member of some collection to the attributes of the totality of that collection.

Computable A system is computable if there is an effective procedure to calculate a solution.

Concept (in Frege's terminology) A function that maps objects to truth-values. For example, the concept of being a horse maps a horse, calls it x, to truth (because x is a horse) but a cow, y, to falsity (because y is not a horse).

Conclusion A proposition that one wants to uphold.

Conditional proof (CP) A strategy in natural deduction for arguments with a conditional conclusion. It proves an argument by first assuming the antecedent of the conclusion, working out its consequent and then discharging the assumption itself.

Conjunction A conjunction is true only when both components (conjuncts) are true.

Conjunctive inference An argument involving conjunction. The following are valid forms: p, q, therefore p & q (composition); p & q, therefore p (decomposition).

Constructive dilemma A dilemma that argues from how the horns are related to how the entailments are related. The general form is: $p \vee q, p \rightarrow r, q \rightarrow s \,/\, \therefore r \vee s$

Containment A categorical syllogism is valid if and only if the premises contain the conclusion, i.e. the conclusion obtains under the conditions that make the premises true.

Contingent statements Statements that are true in one world but false in another.

Contradiction A statement that is always false. Contrast 'tautology'.

Contradictory The relation that propositions have exactly opposite truth-values.

Contraposition A combined operation of obverting a proposition, then converting it, and then obverting it again. The resultant proposition takes the form of having non-P in the subject position and non-S in the predicate position.

Contrary The relation that two propositions cannot both be true but can both be false.

Conversion An operation to form propositions by interchanging the subject and the predicate of an original proposition.

Deduction An argument in which the truth of the premises completely determines the truth of the conclusion given that it is a valid argument.

Definiendum An item to be defined.

Definiens That which defines a term.

Destructive dilemma A dilemma that argues backwards from what the entailments are *not* to what the horns are not. The general form is: $p \rightarrow r, q \rightarrow s, \neg r \vee \neg s \,/\, \therefore \neg p \vee \neg q$

Dilemma An argument form that has two horns: p and q, each entailing some results.

Disjunction A disjunction is false only when both components (disjuncts) are false.

Disjunctive syllogism A syllogism involving disjunction. A valid argument form is $p \vee q, \neg p, \therefore q$. An invalid form is $p \vee, p, \therefore \neg q$

Distribution (in categorical logic) A term is distributed if every member of the class it refers to is exhausted by the proposition containing it.

Division The fallacy of division is formed when (a) one reasons mistakenly from the attributes of a whole to the attributes of one of its parts, and (b) one reasons mistakenly from the attributes of a totality of a collection of entities to the attributes of the individual entities themselves.

Effective A method of proof is effective if every argument can be determined, within a definite number of steps, to be either valid or invalid.

Emotive words Expressions that usually arouse particular feelings or judgements.

Empty names Names that do not have a referent as they depict.

Equivalence Two propositions are equivalent if they always share the same truth-value.

Equivocation The fallacy of equivocation occurs when the same word or phrase is used with two or more meanings, deliberately or accidentally, in the formulation of an argument.

Existential import A proposition has existential import if it is typically used to assert the existence of objects.

Fallacy of affirming the consequent An invalid argument in the following form: $p \rightarrow q, q, \therefore p$.

Fallacy of denying the antecedent An invalid argument in the following form: $p \rightarrow q, \neg p, \therefore \neg q$.

False cause A fallacy in which one treats as the cause of a thing that which is not really the cause of that thing.

False dichotomy or false dilemma An argument commits the fallacy of false dichotomy or false dilemma if it presents two alternatives as exhaustive, when in fact other possibilities exist.

Figure The pattern indicating the location of the middle term in a categorical syllogism.

Function (in mathematics and logic) An operation that maps a certain object to another.

Gambler's fallacy The reasoner falsely assumes that the history of outcomes will affect future outcomes when in fact the events are independent of each other.

Generalization The reverse process of instantiation.

Groupthink A phenomenon studied in social psychology and organizational behaviour in which a group values harmony and coherence over accurate analysis and critical evaluation, thus resulting in an irrational or dysfunctional decision-making outcome.

Hasty generalization When one moves too carelessly or quickly from a single case to an indefensibly broad generalization.

Identity A two-place predicate denoting a relation, often expressed in English as 'is identical to', 'is the very same thing as', or simply 'is', etc.

Idiosyncratic expressions Jargon or expressions with unusual usage deviating from ordinary usage without explicit definition.

Incomplete expression An expression without a specified domain.

Inconsistence Propositions cannot be true together.

Induction An argument in which the conclusion is likely to be true although it can be false even when the premises are true.

Instantiation Construct a model for quantification such that, instead of using variables and quantifiers, we now express formulas solely in terms of individual constants.

Irrelevant conclusion When the premises miss the point, purporting to support one conclusion while in fact supporting or establishing another.

Logic The study of the methods and principles used to distinguish between good and bad reasoning.

Logical analogy A method of arguing for the validity (or invalidity) of an argument by constructing an argument of the same logical structure but which is obviously valid (or invalid).

Logicism The project to reduce arithmetic completely to logic championed by Gottlob Frege (1948–1925).

Major premise The premise containing the major term in a categorical syllogism.

Major term The predicate of the conclusion in a categorical syllogism.

Many-valued logics Logical systems which accept that a proposition may not be either true or false but have other values; for example, intuitionist logic accepts 'indeterminate' propositions, free logic accepts 'truth-valuelessness', fuzzy logic accepts degrees of truth.

Material implication A material implication is false only when the antecedent is true, but the consequent is false.

Middle term The term that appears only in the premises of a categorical syllogism.

Minor premise The premise containing the minor term in a categorical syllogism.

Minor term The subject of the conclusion in a categorical syllogism.

Misplacing the burden of proof When the burden of proving a point is placed on the wrong side. Generally, the burden of proof is on the person whose views go against common sense.

Mixed hypothetical syllogisms Arguments that involve material implications and some other forms of proposition.

Modus ponens A valid argument in the following form: $p \rightarrow q$, p, q.

Modus tollens A valid argument in the following form: $p \rightarrow q$, $\neg q$, $\therefore \neg p$.

Mood Presents the categorical proposition types of a categorical syllogism in the order of the major premise, the minor premise and the conclusion.

Necessary condition 'p is the necessary condition of q' means whenever there is no p, there is no q.

Necessary statements Statements that are always true or always false.

Negation (in propositional logic) A proposition and its negation cannot both be true or false at the same time.

Normative Concerning values or evaluations.

N-place predicate A predicate that connects n number of parties.

Obscurity An expression is obscure when it lacks a core meaning.

Obversion An operation to form propositions by changing the quality of a categorical proposition and replacing the predicate term by its complement (namely, changing P into non-P).

Paradox A puzzle that has apparently reasonable assumptions yet generates absurd consequences.

Premise A reason to support the belief in the conclusion.

Proposition Anything the content of which is capable of being true or false. It is a logical notion.

Propositional function An expression composed of a predicate and a variable; it has no truth-values and must be either bounded by a quantifier (universal or existential) or substituted with an individual constant to become truth-evaluable.

Pure hypothetical syllogism (chain argument) A valid argument that involves only material implications. It has the following form: $p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$.

Red herring Changing the subject, diverting attention away to other issues.

Reductio ad absurdum A reasoning strategy in which we construct an argument in such a way that it leads to a logical contradiction, and we infer from this that at least one of the premises of this argument is false.

Reference The relation between language and reality.

Reference/extension/denotation Objects picked up by an expression.

Reflexive relation A relation that bestows upon itself; in symbols, if a relation *R* is reflexive and *a* is any member of a domain, then *Raa* always holds. Examples: identity, 'is equal to'. Counter-example: love.

Rhetorical question An implicit proposition presented in the form of a different mood of speech, e.g. question or exclamation.

Scope of quantification The domain or extent to which a quantifier governs.

Sense/intension/connotation Criteria or reasons by virtue of which objects are picked up by an expression. In particular, Frege (1892) conceived sense as modes of presentation to the mind or modes of determination of reference. Intension is technically defined by Carnap (1956) as a function from possible worlds to truth-values. Connotation is used by Mill (1872) to mean the properties shared by the set of objects a term denotes.

Sentence A string of symbols that is complete in grammatical structure and meaning. It is a linguistic item.

Singular proposition A proposition asserting that a particular individual has a specified attribute.

Slippery slope argument A small concession is presented as having potentially catastrophic consequences.

Sound argument An argument is sound if it is a valid argument and the premises are all true.

Soundness (of a logical system) A logical system is sound if and only if it does not allow a deduction beginning with true propositions to end up with false ones.

Statement Something that expresses a belief. It is a mental entity.

Straw man fallacy A fallacy that occurs when one levels one's arguments against a crude caricature of one's opponent's views.

Subalternation If P is true, the Q is true, but not *vice versa*.

Sub-argument A simple argument within a more complex argument to support its conclusion.

Subcontrary The relation that two propositions can both be true but cannot both be false.

Sufficient condition '*p* is the sufficient condition of *q*' means whenever *p* exists, *q* exists.

Syllogism An argument that has exactly two premises and one conclusion.

Symmetric relation A relation that if an individual *a* is related to another individual *b*, then *b* is related back to *a* in the same way; in symbols, *Rab* implies *Rba*. Example: 'being a sibling of'. Counter-example: 'being the boss of'.

Synthetic statement Any statement that is not analytic.

Tautology A statement that is always true.

Transitive relation A relation that can be carried forward; in symbols, *Rab* and *Rbc* implies *Rac*. Examples: 'larger than', 'older than'. Counter-example: love.

Truth Whatever it is.

Truth-function A function that maps objects according to their truth-values.

Truth-functional relation A relation in which the truth-value of the combined sentence is *completely determined* by the truth-values of the component sentences.

Truth-functionally equivalent Two propositions are truth-functionally equivalent when they have exactly the same truth-value in every possible truth-value assignment.

Truth-table A complete list of all possible truth-value assignments of a proposition.

Truth-value True or False.

Truth-value assignment The truth-value assigned to a complex proposition depending on the truth-values of its constituent atomic propositions.

Tu quoque (you too) A special type of *ad hominem* argument, in which you attempt to discredit your opponent's views by pointing out that he or she does not always act on them.

Vagueness An expression is vague if it has a core meaning though does not have a clear boundary.

Validity An argument is valid if and only if the conclusion follows the premises. It also means that the conclusion cannot be false if all the premises are true.

Well-formed formula Roughly the same as a proposition, i.e. something that can be either true or false.

Taking it further

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Solutions

Exercise 1.1: Identifying arguments

Premise and conclusion indicators are highlighted in bold.

1 A prince ... must imitate the fox and the lion, **for** the lion cannot protect himself from traps, and the fox cannot defend himself from wolves. One must **therefore** be a fox to recognize traps, and a lion to frighten wolves.

The lion cannot protect himself from traps.

The fox cannot defend himself from wolves.

A prince must recognize traps and frighten wolves. (Implicit assumption reconstructed from text)

Therefore, a prince must imitate the fox and the lion.

2 **Since** happiness consists in peace of mind, and **since** durable peace of mind depends on the confidence we have in the future, and **since** that confidence is based on the science we should have of the nature of God and the soul **it follows that** science is necessary for true happiness.

Happiness consists in peace of mind.

Peace of mind depends on confidence in the future.

Confidence is based on science.

Therefore, science is necessary for happiness.

3 Liberty means responsibility. **That is why** most men dread it.

Liberty means responsibility.

Most men dread responsibility. (Implicit assumption not mentioned in the text)

Therefore, most men dread liberty.

4 'And how do you know that you're mad?' '**To begin with,**' said the Cat, 'a dog's not mad. You grant that?' 'I suppose so,' said Alice. 'Well, then,' the Cat went on, 'you see, a dog growls when it's angry, and wags its tail when it's pleased. Now I growl when I'm pleased, and wag my tail when I'm angry. **Therefore** I'm mad.'

A dog is not mad.

A dog growls when it is angry and wags its tail when it is pleased.

I do not growl when I am angry or wag my tail when I am pleased. (Indeed, on the contrary, I growl when I am pleased and I wag my tail when I'm angry.)

Therefore, I am mad.

5 If a man says, 'I love God', and hates his brother, he is a liar; **for** he that loves not his brother whom he has seen, how can he love God whom he has not seen?
A man has seen his brother.

A man has not seen God.

If a man does not love what he has seen, then he cannot love what he has not seen. (Implicit premise)

Therefore, a man who does not love his brother cannot love God. (Implicit sub-conclusion)

Therefore, if a man says, 'I love God', and hates his brother, he is a liar.

[Note: This example shows that a conclusion can be a complex sentence as well! Implicit premises and the sub-conclusion are reconstructed from the text with the rhetorical complex question. The question is rhetorical because it leads to an answer and is not meant to be an open-ended question. It is complex because it has assumed something to be the case.]

6 This is an open question. Suggest an argument and justify your reconstruction with techniques introduced in this chapter, such as the identification of premise or conclusion indicators.

Exercise 1.2: Testing concepts of validity

1 i F; ii F; iii T; iv F; v T

2 i–c; ii–d; iii–h; iv–g; v–a; vi–b; vii–f; viii–e

Exercise 2.1: Spotting problems of meaning

1 **Vagueness and ambiguity**: 'Success' is a vague and ambiguous term. It is vague in that there are possibly different degrees of success and the sentence does not specify which. It is ambiguous in that there are so many different ways to define success. Take social movement as an example. It is possible that the protesters regard overthrowing the government as success. Alternatively, they may regard arousing public awareness of the tyranny of the government is itself a goal. Depending on the definition, one may evaluate a movement as failure in that it does not overthrow the existing government but is indeed suppressed by it. Yet the same result may be regarded as a success in which the protesters may claim that the public is more aware of unjust suppression the government put on its opponents. Without this awareness the eventual success of overthrowing the government may never come.

Incomplete expression: The most prominent problem with this sentence, however, is that it is incomplete. What counts as the 'right' thing and the 'right' time? They could mean anything and anytime! If an undesired outcome occurs, do we say it is failure or that it simply is not the right time to act or to judge? Do we define 'right' in terms of success, such that whenever we fail we just excuse it as not doing the right thing at the right time? If that is the case, we are simply running around in circles, for the definitions of 'right' and 'success' are now circular.

2 **Emotive words**: The utterance avoids using the word 'die', though indeed it allures to the same fact as disappearance. Saying that veterans don't die, however, may trigger an

emotive response in the audience (possibly the family and colleagues of the veterans) amidst the grief of their losses.

3 **Obscurity**: It is not clear what is meant by 'all is vanity'. Indeed, words like 'vanity' make sense only if they are contrasted with opposites; for example, wealth and reputation is vanity when compared to true love and friendship. If literally everything is vanity, then 'vanity' itself will lose meaning too.

4 **Ambiguity**: The 'good' in 'be good' and 'good things' may mean different things. 'Be good' means doing something morally good. Whereas the 'good things' in 'good things will come to you' suggests benefits, welfare, interests, fortunes, luck, etc. There are many cases in which morally good people do not receive any good treatment or rewards for their good deeds. Hence, given the ambiguity in the term 'good', the sentence may well be false. There is however another problem with this sentence too. Let us discuss below.

Vagueness: Both the terms 'good' and 'eventually' are vague. There are various degrees of goodness, in whatever sense implied as discussed above. The word 'eventually' also signifies an indefinite period of time for the judgement to be made. We can always assert that good things will come even though they have not come yet. This justifies an indefinite wait. Therefore, eventually the sentence is unfalsifiable at any given point of time because of the vagueness of these expressions.

5 **Ambiguity**: This phrase is found on notice boards. It can be interpreted in two different ways: (1) No-smoking seats are available, meaning that there are seats reserved for non-smokers. (2) No smoking-seats are available, meaning that no seats are available for smokers. These two interpretations are indeed distinct in content and address different subjects. Having non-smoker seats, as in case (1), does not imply that there are no designated seats for smokers. So if you are a smoker, you are still welcome to enter. Whereas if you are a non-smoker, the notice also does not reassure you that you are free of smokers around you. Hence, you may still have the problem of passive smoking! If the second interpretation

is the case, that means there are no seats for smokers. If you are a smoker, then you should not enter. If you are a non-smoker, you can be assured that you won't have anyone smoking around you. These two interpretations thus convey completely different messages and imply different effects on their audience.

6 **Incompleteness:** Honesty is the best strategy, but 'best' for what purpose? In what contexts? And among what alternatives? Without specifying the framework of reference, it is not clear what the statement is about. This also makes the assertion impossible to refute, because we don't even know what counts as an instance of its success (i.e. situations in which this sentence is true).

7 **Ambiguity:** What is 'Western democracy'? Many countries are generally and vaguely regarded as the West or the Western countries. Europe is different from the USA. Germany is different from France or the UK. Moreover, within a country there could also be different systems of democracy practised in different periods of time. Is there really one model to be signified by the term 'Western democracy'? It seems the answer would be negative. So the term 'Western democracy' is ambiguous. Similarly, the term 'real democracy' is also ambiguous. It may mean many different things! Democracy may just mean any government form in which the majority rules. Or it may require the fulfilment of certain political ideals and values. If so, what specifically are those ideals and values behind democracy? It is sometimes said that democracy is government of the people, by the people, and for the people. Yet each of these phrases has different connotations, not to mention that these qualities could in principle exist independently.

Emotive words: The use of the word 'real' also suggests some positive mood as if this other type of democracy is the better one, when in fact probably we still need to see what this democracy means and decide whether it is indeed better for the regime involved in the context.

8 **Idiosyncrasy:** In common understanding of words like 'I' and 'the world', I and the world are distinct. I am a part

of the world, and the world is bigger than I am. The world comprises of many things outside of me. In fact, this is how we make sense of an 'inner' realm and an 'external' world. In the above quote, however, philosopher Ludwig Wittgenstein made a rather odd claim. This is a solipsistic view of the world. No matter what it is and irrespective of whether it is true or not, since it is something quite remote from ordinary use of the terms, it is idiosyncrasy and the philosopher has the burden to explain his idea.

Exercise 2.2: Evaluating definitions

1 **Not specifying the essential attributes:** An oil painting should be painted in oils. This is a classification based on a painting medium. However, this definition does not mention the medium at all but just the material to paint on and tools used. It does not capture the distinguishing feature of the type.

Too narrow: An oil painting does not have to be drawn on a canvas or with a brush. It can be painted on walls, wood, with knives or other tools.

Too broad: Other types of art can be drawn on a canvas with a brush, e.g. acrylics. Even watercolours can be painted on canvas though some preparation is needed.

2 **Obscure and figurative:** When we are sure of things, we have knowledge, reason, evidence; we do not need faith. People need faith only when they have no such certainty yet they still want to believe and uphold something. We need faith exactly when we do not know whether the substance we hope for will come true, or when we do not possess the evidence of the things in question. This definition seems to twist what we need faith *for*, or what we lack, to what faith *is*. We may want faith to satisfy a need of ours. Yet whether faith can do so is an arguable question. This definition uses obscure and figurative language to mislead us away from seeing the true nature of faith but just our own wish.

3 **Too broad:** What people and the society regard as meaningful may not be good. Not to mention that sometimes what is good for an individual is not necessarily regarded as good or meaningful by the majority in the society. Meaningful for whom is the question.

Too narrow: The definition focuses on what people perceived as good for them. This is the subjective side of the matter and is very individualistic. However, our well-being is also objective because we share similar biology and general psychological makeup as human beings. So the definition seems to be too one-sided.

4 **Ambiguous and obscure:** This definition is literally more difficult to understand than the term to be defined. The author employed abstract and seemingly technical and highly theoretical jargon such as homogeneity, heterogeneity, dissipation of motion to define evolution. Even the meaning of terms like integration of matter may not be clear in this context. It is also unclear what those adjectives mean when they apply to the abstract terms like these. Audience is probably left dazzled and more puzzled rather than getting to understand more about evolution.

5 **Too narrow:** It can be argued that love does not only aim at reproduction. Infertile couples can still be deeply in love, not to mention homosexual partners and people with other sexual orientations. As technology advances, love and sex are different matters: contraception, artificial reproduction, surrogacy, etc. are all common practices.

Not capturing essential attributes: Love is more importantly a relationship; it relates persons. It is endowed with emotions, feelings and passions so strong that it can lead to quite extreme actions. It can result in very fulfilling lives or cause very destructive consequences. All these seem worth considering as essential to love and should be included in a definition about it.

Should avoid being negative: The choice of words like 'tricking' adds a negative connotation to the definition.

It almost sounds as if the ones who assent to this definition are likely to have some regret or bad experience about love, family or reproduction. Yet indeed, all these matters can be neutral and different people may have very different takes on them.

6 **Circular:** 'Meaning is what is explained by the explanation of meaning' sounds painfully circular. For it does not explain what it is to give an explanation of meaning, except that an explanation of meaning explains a meaning. The situation can be improved if the author independently explains what it is to give an explanation of meaning, say, by giving examples, illustrations, categorizations, so on and so forth. In other words, just explain more about what is an explanation of meaning!

It does not mean what is quoted about has no insight. Wittgenstein's point is indeed in the second half of the quote. He is battling against the endeavour of giving a single generalized account of meaning when, he observed, indeed that language is used in very diverse ways. There are way too many occasions possibly and actually regarded as explanations of meaning; there is perhaps **no essence**, so we cannot really give meaning a definition.

7 **Too broad:** Thought or feeling can be expressed in words of many formats. Yet not all forms and structures of words are regarded as literature, not to mention that not all literature types are poetry. Poetry also has certain requirement on the rhythm and the sound of the arrangement of words.

Ambiguous and figurative: Adjectives like 'lofty', 'imaginative' are vague. 'Lofty' is figurative when applied to thoughts.

8 **Figurative:** Religion is a daughter of hope and fear… Obviously, religion is a social institution and cannot literally be 'a daughter'. What the author means is probably that religion exists because people hope for a better world yet are in fear of the unknown.

9 **Too narrow:** Under this definition, social science is not a science because it is not just about the physical or material

world. Moreover, it is possible that some experimentation is not allowed in social science for ethical reasons.

10 **Circular:** Obvious. If people do not already understand what it is to be just, then they would not get more knowledge about justice from this definition either.

Overall moral: It is perhaps often difficult to get a perfect definition of things. Explanation and clarification is not restricted to giving definitions, though. Moreover, there are other types of definition like enumeration, ostensive definition, etc.

Exercise 3.1: Informal fallacies (1)

1 **Slippery slope.** The argument can be reconstructed as follows:

a If we legalize abortion, we will end up with child pornography etc.

b We must not end up with child pornography.

c We must not legalize abortion.

The first premise is (presumably) a false causal claim.

2 **False dilemma or false dichotomy.** Approving drug taking and approving the war on drugs are not the only two possible positions.

3 **Equivocation due to ambiguity.** There are two senses of the word 'desirable'. One sense is that an object *can be desired* or is *capable of being desired*. The other sense is that an object is *worthy of being desired*. Mill seems to be taking an object being desirable in the first sense as grounds for its desirability in the second sense.

4 **Composition.** The parts being non-accidental do not imply that the whole entity cannot be accidental. There may not be one purpose for the whole but only many purposes for the components. It is like everyone is loved by someone does not imply that everyone is loved by the same person.

5 **False dilemma and emotive words.** Can we not choose to live?!

6 **Equivocation due to vagueness.** Different comparison classes are linked with the word 'small' in its two occurrences.

7 **Division**. What is good for all does not mean it is good for an individual. There may be inequality such that some may suffer seriously while others gain a lot and then the country may claim it is for the common good.

8 **Slippery slope**. In fact, this is the sorites paradox, a paradox about vagueness.

Exercise 3.2: Informal fallacies (2)

1 **Appeal to inappropriate authority**: The weight of air is subject to empirical discovery, not philosophical discussion. Aristotle was a genuine scientist and a reputable authority in his own time, although he made numerous scientific errors. The argument is fallacious because it appeals to an authority who wrote nearly two thousand years ago and is no more an authority in modern science.

2 **Appeal to ignorance.** It is fine to keep an open mind about the discovery of America. However, if the argument is that an event did happen (i.e. America was discovered by Africans) just because we cannot disprove it, then the argument commits the fallacy of appeal to ignorance. An unlimited number of previously revealed discoveries may be fallaciously defended in this way.

3 There are two fallacies. First, the argument ***ad hominem*, circumstantial**: it is wrong to claim that the logicians' view is faulty just because they want to keep themselves employed. Second, the appeal to **common practice**: the view represented by the majority of students, parents or teachers may not represent the correct view.

4 This is the ***tu quoque*** fallacy. It can be reconstructed as follows:

a If you commit fallacies in logic, then you cannot detect fallacies in my logic.

b You commit fallacies in logic.

c Therefore you cannot detect fallacies in my logic.

The first premise is false.

5 Argument *ad hominem*, abusive: The argument assumes that women cannot be good. Equivocation: The criteria to judge whether a journalist is bad is not the same as that to judge whether a woman is bad.

6 Misplacing the burden of proof. A is the one who makes this claim against common sense and so A should be the one to prove it.

7 Accident. Students using textbooks in examinations is not a proper instance of the general rule of freedom of reference.

8 Begging the question. A cause is by definition what causes an effect on something.

Exercise 4.1: Translating into categorical propositions

1 No orchids are fragrant plants. (E)

2 Some people worth meeting are not people worth having as a friend. (I)

3 All people who know their limitations are happy people. (A)

4 All persons who can use the front door are members. (A)

5 All occasions on which he is asked to say a few words are occasions on which he talks for hours. (A)

6 No students of mine are failing persons. (E)

7 All people who live in the dorms are people who cannot own a car. (A). Or: No people who live in the dorms are people who own a car. (E)

8 All the tests George takes are the tests George fails. (A)

9 All the things you lose are your chains. (A)

10 All people who pass the course are people who pass this test. (A) Or: No people who fail this test are people who pass the course. (E)

11 Some home movies are boring things. (I)

12 Some part of Michael Jackson's face is not an original part. (O)

Exercise 4.2: Using Venn diagrams to represent categorical propositions

What categorical propositions do these diagrams show?

1 No A is B (E-proposition)

2 All A is B (A-proposition)

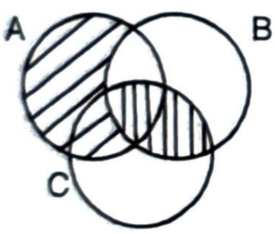
3 Some A is B (I-proposition)

4 Some A is not B (O-proposition)

5 This diagram indicates that some object is the member of all A, B and C. So, it entails the truth of all the following I-propositions: Some A is B, Some B is C, Some A is C.

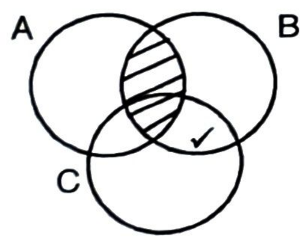
Exercise 4.3: Testing validity using Venn diagrams

1 I



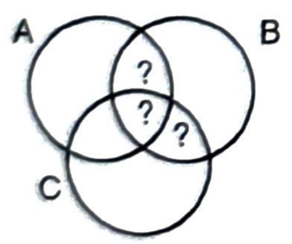
Valid

II



Valid

III



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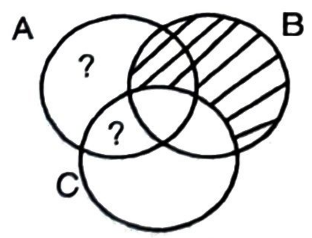
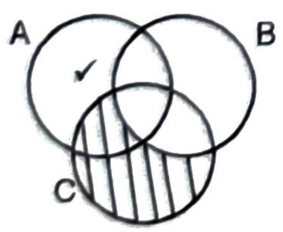
IV



Invalid

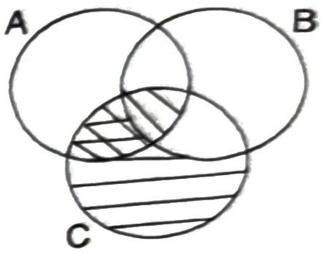
V



Valid

2 *Note: Space does not permit conclusions to be circled here so the requirement is only described in words. However, you should try to circle the conclusion when you do the exercise. Premise/conclusion indicators are put in bold to help you identify the premises and the conclusion correctly.

i No television stars are certified public accountants, but all certified accountants are people of good business sense; **it follows that** no television stars have good business sense.



No television stars are certified public accountants.

All certified public accountants are people of good business sense.

Therefore, no television stars have good business sense.

Let A = television stars, B = people of good business sense, C = certified public accountants

All C is B. No A is C.Therefore, no A is B.

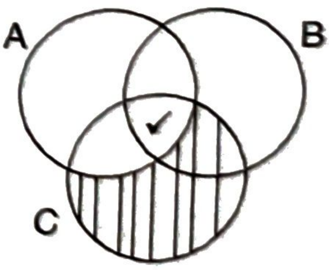
The conclusion requires that both ABC and $AB\overline{C}$ are empty. However, $AB\overline{C}$ is not shaded out, i.e. it is not sure that this area is empty. The argument is invalid.

ii All juvenile delinquents are maladjusted individuals, and some juvenile delinquents are products of broken homes; **hence** some maladjusted individuals are products of broken homes.

All juvenile delinquents are maladjusted individuals.

Some juvenile delinquents are products of broken homes.

Therefore, some maladjusted individuals are products of broken homes.



Let A = maladjusted individuals,
B = products of broken homes,
C = juvenile delinquents.

All C is A. Some C is B. Therefore, some A is B.

The conclusion requires that either ABC or $AB\overline{C}$ has objects.

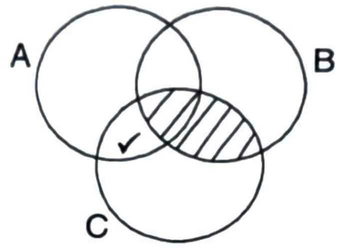
ABC has objects. Hence the argument is valid.

iii No intellectuals are successful politicians, because no shy and retiring people are successful politicians, and some intellectuals are shy and retiring.

No shy and retiring people are successful politicians.

Some intellectuals are shy and retiring people.

Therefore, no intellectuals are successful politicians.



Let A = intellectuals, B = successful politicians, C = shy and retiring people.

No C is B, some A is C; therefore, no A is B.

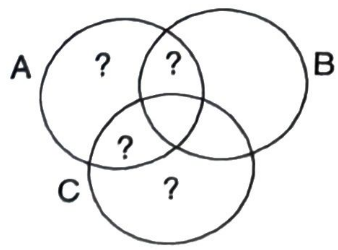
The conclusion requires that both ABC and $AB\overline{C}$ are empty. $AB\overline{C}$ is not surely empty. Therefore, the argument is invalid.

iv Some Christians are not Methodists, for some Christians are not Protestants, and some Protestants are not Methodists.

Some Christians are not Protestants.

Some Protestants are not Methodists.

Therefore, some Christians are not Methodists.



Let A = Christians, B = Methodists, C = Protestants

Some A is not C. Some C is not B. Therefore, some A is not B.

The conclusion requires that $A\overline{B}C$ or $A\overline{B}\overline{C}$ has some objects. The Venn diagram shows however that none of these areas definitely has objects. Hence, the argument is invalid.

Exercise 4.4: More validity tests

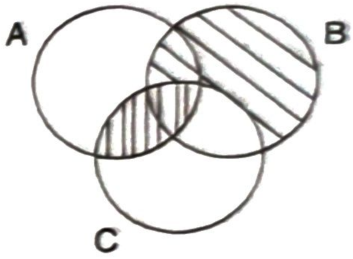
*Note: Space does not permit conclusions to be circled here so the requirement is only described in words. However, you should try to circle the conclusion when you do the exercise. Premise/conclusion indicators are put in bold here to help you identify the premises and the conclusion and translate correctly.

1 **Because** no weaklings are true liberals, and all labour leaders are true liberals; **so**, no weaklings are labour leaders.

No weaklings are true liberals.

All labour leaders are true liberals.

Therefore, no weaklings are labour leaders.

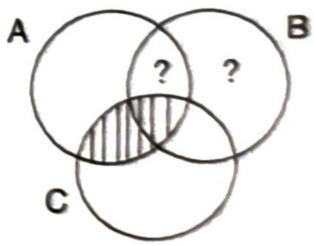


Let A = weaklings, B = labour leaders, C = true liberals

No A is C. All B is C. Therefore, no A is B.

The conclusion requires that both ABC and $AB\bar{C}$ are empty. They are indeed empty. Hence, the argument is valid.

2 All centaurs are mammals, **for** some mammals are not horses, and no horses are centaurs.



Some mammals are not horses.

No horses are centaurs.

Therefore, all centaurs are mammals.

Let A = centaurs, B = mammals, C = horses

Some B is not C. No C is A.Therefore, all A is B.

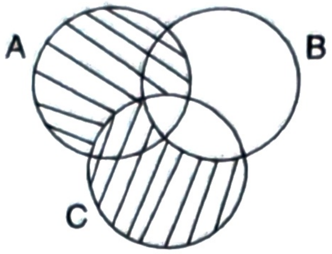
The conclusion requires that both $A\bar{B}\bar{C}$ and $A\bar{B}C$ are empty. $A\bar{B}\bar{C}$ is not definitely empty. Therefore, the argument is invalid.

3 All criminal actions are wicked deeds. All prosecutions for murder are criminal actions. **Therefore**, all prosecutions for murder are wicked deeds.

All criminal actions are wicked deeds.

All prosecutions for murder are criminal actions.

Therefore, all prosecutions for murder are wicked deeds.

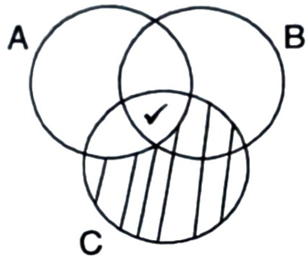


Let A = prosecutions for murder, B = wicked deeds, C = criminal actions

All C is B. All A is C. Therefore, all A is B.

The conclusion requires that both $A\bar{B}\bar{C}$ and $A\bar{B}C$ are empty. They are indeed empty. Hence, the argument is valid.

4 Some reformers are fanatics, so some idealists are fanatics, since all reformers are idealists.



Some reformers are fanatics.

All reformers are idealists.

Therefore, some idealists are fanatics.

Let A = idealists, B = fanatics, C = reformers

Some C is B. All C is A. Therefore, some A is B.

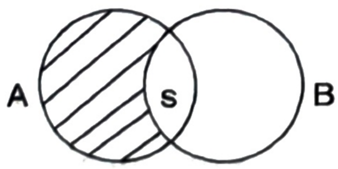
The conclusion requires that ABC or $AB\bar{C}$ has some object.

There is an object in ABC. Hence, the argument is valid.

5 All men are mortal. Socrates is a man. Therefore, Socrates is mortal.

This question tests the treatment of singular terms. There are two ways to handle them. Either is acceptable.

<u>Method 1: represent Socrates as a particular</u>



All men are mortal beings.

Socrates is a man.

Therefore, Socrates is a mortal being.

Let A = men, B = mortal beings, s = Socrates

All A is B. s is A. Therefore, s is B.

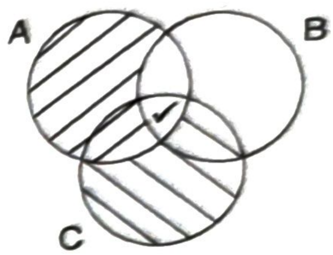
The conclusion requires that s is in B. Now s is in the area of AB. Hence, the argument is valid.

<u>Method 2: represent 'Socrates' as a singleton set</u>

All men are mortal beings.

All objects that are Socrates are men and some object that is Socrates is a man.

Therefore, all objects that are Socrates are mortal beings and some object that is Socrates is a mortal being.



Let A = men, B = mortal beings, C = object that is Socrates

All A is B. All C is A and some C is A.

Therefore, All C is B and some C is B.

The conclusion requires that $A\bar{B}C$ and $\bar{A}\bar{B}C$ are empty, and there is an object in ABC or $\bar{A}BC$.

$A\bar{B}C$ and $\bar{A}\bar{B}C$ are empty and there is indeed an object in ABC.

Therefore, the argument is valid.

6 She told me that she had a very simple attitude toward her students which was in fact no different from her feelings about people in general. That was, all her life she'd spoken only to people who were ladies and gentlemen. Since none of the students of 9D were ladies and gentlemen, she never spoke to them, never had, and never would.

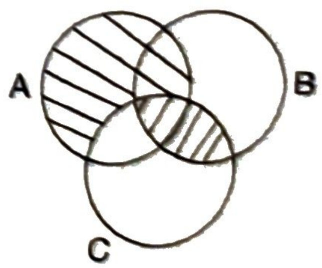
This question tests the skill to extract only relevant information and reformulate the argument into a standard form.

Step 1: extract relevant information, identify the conclusion and premises

All her life she'd only spoken to people who were ladies and gentleman.

None of the students of 9D were ladies and gentlemen.

Therefore, she never spoke to students of 9D.



Step 2: write into standard form

All people she had spoken to were ladies and gentleman.

No students of 9D were ladies and gentlemen.

Therefore, no people she had spoken to were students of 9D.

Let A = people she had spoken to, B = students of 9D, C = ladies and gentlemen

All A is C. No B is C. Therefore, No A is B.

The conclusion requires that both ABC and $AB\bar{C}$ are empty. They are indeed empty. Hence, the argument is valid.

7 **It is not true that** less destructive nuclear weapons are not dangerous, **because** less destructive nuclear weapons make it easier for a nuclear war to begin and anything that starts a nuclear war easy is dangerous.

This question is about negatives.

Step 1: identify conclusion and premises

Less destructive nuclear weapons are things that make it easier for a nuclear war to begin.

Anything that makes it easier for a nuclear war to begin is a dangerous thing.

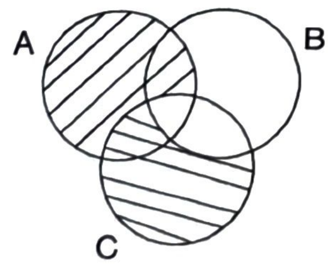
Therefore, it is not true that the less destructive nuclear weapons are not dangerous things.

Step 2: write into standard form

All less destructive nuclear weapons are things that make it easier for a nuclear war to begin.

All things that make it easier for a nuclear war to begin are dangerous things.

Therefore, all less destructive nuclear weapons are dangerous things.



Let A = the less destructive nuclear weapons, B = dangerous things,

C = things that make it easier for a nuclear war to begin

All A is C. All C is B. Therefore, all A is B.

The conclusion requires that $A\bar{B}\bar{C}$ and $A\bar{B}C$ to be empty.

They are both empty. Therefore, the argument is valid.

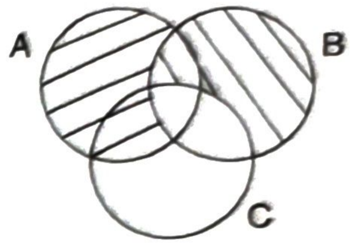
8 Whenever there's a challenge there's opportunity, so we are grateful whenever there is a challenge, **because** we are grateful whenever there is opportunity.

This question is about the use of parameters.

All occasions of challenge are occasions of opportunity.

All occasions of opportunity are occasions of gratitude.

Therefore, all occasions of challenge are occasions of gratitude.



Let A = occasions of challenge, B = occasions of opportunity, C = occasions of gratitude

All A is B. All B is C. Therefore, all A is C.

The conclusion requires that $AB\bar{C}$ and $A\bar{B}\bar{C}$ are both empty.

They are both empty. Therefore, the argument is valid.

Exercise 4.5: Testing validity using the rule method

1 The major term is C, the minor term is B, the middle term is A.

> All A^D is C. (Major premise)
>
> All A^D is B. (Minor premise)
>
> ∴ All B^D is C. (Conclusion)

Mood and figure: AAA-3. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 3: B is distributed in the conclusion but not in the premise.

Therefore, the argument is invalid.

2 The major term is B, the minor term is A, the middle term is C.

> Some C is not B^D. (Major premise)
>
> Some A is not C^D. (Minor premise)
>
> ∴ Some A is not B^D. (Conclusion)

Mood and figure: OOO-1. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 1: The conclusion is negative but both premises are negative; so the numbers of negative propositions in the premise and the conclusion do not agree.

Therefore, the argument is invalid.

3 The major term is A, the minor term is C, the middle term is B.

All A^D is B. (Major premise)

No B^D is C^D. (Minor premise)

∴ Some C is not A^D. (Conclusion)

Mood and figure: AEO-4. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 4: A particular conclusion cannot be derived from universal propositions alone.

Therefore, the argument is invalid.

4 The major term is A, the minor term is B, the middle term is C.

No A^D is C^D. (Major premise)

All B^D is C. (Minor premise)

∴ No B^D is A^D. (Conclusion)

Mood and figure: EAE-2. All distributed terms are indicated by ^{D} in the above formulation.

No rules are broken. Therefore, the argument is valid.

5 The major term is B, the minor term is A, the middle term is C.

All B^D is C. (Major premise)

Some A is C. (Minor premise)

∴ Some A is B. (Conclusion)

Mood and figure: AII-2. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 2: The middle term is not distributed at least once.

Therefore, the argument is invalid.

6 The major term is B, the minor term is A, the middle term is C.

All B^D is C. (Major premise)

All A^D is C. (Minor premise)

∴ Some A is B. (Conclusion)

Mood and figure: EAI-2. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 2: The middle term is not distributed at least once.

Break Rule 4: A particular conclusion cannot be derived from universal premises.

Therefore, the argument is invalid.

7 The major term is B, the minor term is A, the middle term is C.

All C^D is B. (Major premise)

Some A is C. (Minor premise)

∴ No A^D is B^D. (Conclusion)

Mood and figure: AIE-1. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 1: The conclusion is negative yet there is no negative premise.

Break Rule 3: Both A and B are distributed in the conclusion but not the premise.

Therefore, the argument is invalid.

8 The major term is B, the minor term is C, the middle term is A.

All B^D is A. (Major premise)

All C^D is A. (Minor premise)

∴ Some C is not B^D. (Conclusion)

Mood and figure: AAO-2. All distributed terms are indicated by ^{D} in the above formulation.

Break Rule 1: The conclusion is negative yet there is no negative premise.

Break Rule 2: The middle term is not distributed at least once.

Break Rule 4: A particular conclusion cannot be derived from universal premises alone.

Therefore, the argument is invalid.

Exercise 5.1: Drawing truth-tables

1 $p \rightarrow (\neg p \vee p)$

p	p → (¬ p ∨ p)
T	T T F T T T
F	F T T F T F

2 $(p \,\&\, \neg p) \equiv p$

p	(p & ¬ p) ≡ p
T	T F F T F T
F	F F T F T F

3 $p \rightarrow \neg q$

p	q	p → ¬ q
T	T	T F F T
T	F	T T T F
F	T	F T F T
F	F	F T T F

4 $\neg p \vee q$

p	q	¬ p ∨ q
T	T	F T T T
T	F	F T F F
F	T	T F T T
F	F	T F T F

5 $\neg (p \rightarrow q)$

p	q	¬ (p → q)
T	T	F T T T
T	F	T T F F
F	T	F F T T
F	F	F F T F

6 $p \equiv [q \vee (\neg q \rightarrow p)]$

p	q	$p \equiv [q \vee (\neg q \rightarrow p)]$
T	T	TT TT FTTT
T	F	TT FT TFTT
F	T	FF TT FTTF
F	F	FT FF TFFF

7 $p \mathbin{\&} \neg (q \rightarrow r)$

p	q	r	$p \mathbin{\&} \neg (q \rightarrow r)$
T	T	T	TFF TTT
T	T	F	TTT TFF
T	F	T	TFF FTT
T	F	F	TFF FTF
F	T	T	FFF TTT
F	T	F	FFT TFF
F	F	T	FFF FTT
F	F	F	FFF FTF

8 $(p \mathbin{\&} \neg q) \rightarrow r$

p	q	r	$(p \mathbin{\&} \neg q) \rightarrow r$
T	T	T	TFFT TT
T	T	F	TFFT TF
T	F	T	TTTF TT
T	F	F	TTTF FF
F	T	T	FFFT TT
F	T	F	FFFT TF
F	F	T	FFTF TT
F	F	F	FFTF TF

9 $p \to (q \to r)$

p	q	r	$p \to (q \to r)$
T	T	T	TT TTT
T	T	F	TF TFF
T	F	T	TT FTT
T	F	F	TT FTF
F	T	T	FT TTT
F	T	F	FT TFF
F	F	T	FT FTT
F	F	F	FT FTF

10 $(p \to q) \to r$

p	q	r	$(p \to q) \to r$
T	T	T	TT T TT
T	T	F	TT T FF
T	F	T	TF F TT
T	F	F	TF F TF
F	T	T	FT T TT
F	T	F	FT T FF
F	F	T	FT F TT
F	F	F	FT F FF

11 $(p \to q) \vee (r \to \neg q)$

p	q	r	$(p \to q) \vee (r \to \neg q)$
T	T	T	TT T T TFFT
T	T	F	TT T T FTFT
T	F	T	TF F T TTTF
T	F	F	TF F T FTTF
F	T	T	FT T T TFFT
F	T	F	FT T T FTFT
F	F	T	FT F T TTTF
F	F	F	FT F T FTTT

12 $(p \mathbin{\&} q) \rightarrow (\neg q \rightarrow \neg r)$

p	q	r	$(p \mathbin{\&} q) \rightarrow (\neg q \rightarrow \neg r)$
T	T	T	TTT T FTTFT
T	T	F	TTT T FTTTF
T	F	T	TFF T TFFFT
T	F	F	TFF T TFTTF
F	T	T	FFT T FTTFT
F	T	F	FFT T FTTTF
F	F	T	FFF T TFFFT
F	F	F	FFF T TFTTF

13 $(p \equiv r) \rightarrow [(p \rightarrow q) \mathbin{\&} (\neg r \rightarrow \neg q)]$

p	q	r	$(p \equiv r) \rightarrow [(p \rightarrow q) \mathbin{\&} (\neg r \rightarrow \neg q)]$
T	T	T	TTTT TTTT FTTFT
T	T	F	TFFT TTTF TFFFT
T	F	T	TTTF TFFF FTTTF
T	F	F	TFFT TFFF TFTTF
F	T	T	FFTT FTTT FTTFT
F	T	F	FTFF FTTF TFFFT
F	F	T	FFTT FTFT FTTTF
F	F	F	FTFT FTFT TFTTF

Exercise 5.2: Translating truth-functional connectives

1 Let A = Saudi Arabia buys 500 more warplanes, B = Iran raises the oil price, C = Jordan requests more American aid.

(A & B) ∨ C

A	B	C	(A & B) ∨ C
T	T	T	T T T T T
T	T	F	T T T T F
T	F	T	T F F T T
T	F	F	T F F F F
F	T	T	F F T T T
F	T	F	F F T F F
F	F	T	F F F T T
F	F	F	F F F F F

2 Let A = Egypt's food shortage worsens, B = Jordan requests more American aid

$\neg (A \vee B)$

A	B	¬ (A ∨ B)
T	T	F T T T
T	F	F T T F
F	T	F F T T
F	F	T F F F

3 Let A = he has a good lawyer, B = he will be acquitted.

$A \rightarrow B$

A	B	A → B
T	T	T T T
T	F	T F F
F	T	F T T
F	F	F T F

4 Let A = you deceive another human being, B = you coerce another human being, C = you violate another human being's rights

$(A \vee B) \rightarrow C$

A	B	C	(A ∨ B) → C
T	T	T	T T T T T
T	T	F	T T T F F
T	F	T	T T F T T
T	F	F	T T F F F
F	T	T	F T T T T
F	T	F	F T T F F
F	F	T	F F F T T
F	F	F	F F F T F

5 Let A = Britain joined the war in Iraq, B = the USA declared the war in Iraq

A ≡ B

A	B	A ≡ B
T	T	T T T
T	F	T F F
F	T	F F T
F	F	F T F

6 Let A = I see his wound, B = I touch his scar, C = I believe in him

C → (A & B)

A	B	C	C → (A & B)
T	T	T	TT TTT
T	T	F	FT TTT
T	F	T	TF TFF
T	F	F	FT TFF
F	T	T	TF FFT
F	T	F	FT FFT
F	F	T	TF FFF
F	F	F	FT FFF

7 Let A = Argentina mobilizes, B = Brazil will protest to the UN, C = Chile calls for a meeting of the Latin American states, D = Dominican Republic calls for a meeting of the Latin American states

$(\neg C \& \neg D) \rightarrow (A \rightarrow B)$

A	B	C	D	$(\neg C \& \neg D) \rightarrow (A \rightarrow B)$
T	T	T	T	FTFFT T T T T
T	T	T	F	FTFTF T T T T
T	T	F	T	TFFFT T T T T
T	T	F	F	TFTTF T T T T
T	F	T	T	FTFFT T T F F
T	F	T	F	FTFTF T T F F
T	F	F	T	TFFFT T T F F
T	F	F	F	TFTTF F T F F
F	T	T	T	FTFFT T F T T
F	T	T	F	FTFTF T F T T
F	T	F	T	TFFFT T F T T
F	T	F	F	TFTTF T F T T
F	F	T	T	FTFFT T F T F
F	F	T	F	FTFTF T F T F
F	F	F	T	TFFFT T F T F
F	F	F	F	TFTTF T F T F

8 Let A = you have superpowers, B = you get away from the situation, C = you are unharmed.

$A \vee \neg (B \& C)$, or equivalently,
$A \vee (\neg B \vee \neg C)$, $(B \& C) \rightarrow A$, $\neg A \rightarrow \neg (B \& C)$

A	B	C	$A \vee \neg (B \& C)$
T	T	T	TTF TTT
T	T	F	TTT TFF
T	F	T	TTT FFT
T	F	F	TTT FFF
F	T	T	FFF TTT
F	T	F	FTT TFF
F	F	T	FTT FFT
F	F	F	FTT FFF

Exercise 5.3: Common valid argument forms and fallacies

1 Let p = it rains, q = the ground is wet
$p \rightarrow q, p, \therefore q$ valid, *modus ponens*

2 Let p = it rains, q = the ground is wet
$p \rightarrow q, q, p$ invalid, fallacy of affirming the consequent

3 Let p = it rains, q = the ground is wet
$p \rightarrow q, \neg q, \therefore \neg p$ valid, *modus tollens*

4 Let p = it rains, q = the ground is wet
$p \rightarrow q, \neg p, \therefore \neg q$ invalid, fallacy of denying the antecedent

5 Let p = the storm comes, q = the river is flooded, r = the houses are destroyed.
$p \rightarrow q, q \rightarrow r, \therefore p \rightarrow r$ valid, pure hypothetical syllogism

6 Let p = Tom was naughty to play in mud, q = Tom fell into mud by accident.
$p \vee q, \neg p, \therefore q$ valid, disjunctive syllogism

7 Let p = Tom was naughty to play in mud, q = Tom fell into mud by accident.
$p \vee q, p, \therefore \neg q$ invalid, disjunctive syllogism

Exercise 5.4: The truth-table method

1 Let A = she forgot, B = she was able to finish

$\neg$ (A $\vee$ $\neg$ B), $\therefore$ B

A	B	¬ (A ∨ ¬ B)	B
T	T	F T T F T	T
T	F	F T T T F	F
F	T	T F F F T	T
F	F	F F T T F	F

Valid argument. It is because there is no case in which the premises are all true but the conclusion is false. When the premise is true (row 3), the conclusion is also true.

2 Let A = the manager noticed the change, B = the manager approved of the change

¬ A ∨ B, A, ∴ B

A	B	¬ A ∨ B	A	B
T	T	F T T T	T	T
T	F	F T F F	T	F
F	T	T F T T	F	T
F	F	T F T F	F	F

Valid argument. It is because there is no case in which the premises are all true but the conclusion is false. When the premise is all true (row 1), the conclusion is also true.

3 Let A = humans have free will, B = God knows what every human being will do next

A → ¬ B, ¬ A, ∴B

A	B	A → ¬ B	¬ A	B
T	T	T F F T	F T	T
T	F	T T T F	F T	F
F	T	F T F T	T F	T
F	F	F T T F	T F	F

Invalid argument. It is because it has a case in which the premises are all true but the conclusion is false (row 4). This is the fallacy of denying the antecedent.

4 Let A = God exists, B = God is good, C = God tolerates evil

A → B, B → ¬ C, C, ∴ ¬ A

A	B	C	A → B	B → ¬ C	C	¬ A
T	T	T	T T T	T F F T	T	F T
T	T	F	T T T	T T T F	F	F T
T	F	T	T F F	F T F T	T	F T
T	F	F	T F F	F T T F	F	F T
F	T	T	F T T	T F F T	T	T F
F	T	F	F T T	T T T F	F	T F
F	F	T	F T F	F T F T	T	T F
F	F	F	F T F	F T T F	F	T F

Valid argument. It is because there is no case in which the premises are true but the conclusion is false. The only case where the premises are true is row 7 and the conclusion is true.

5 Let A = the MP votes against this bill, B = the MP is opposed to penalties against tax evaders, C = the MP is a tax evader himself

$A \rightarrow B, C \rightarrow B, \therefore A \rightarrow C$

A	B	C	A → B	C → B	A → C
T	T	T	T T T	T T T	T T T
T	T	F	T T T	F T T	T F F
T	F	T	T F F	T F F	T T T
T	F	F	T F F	F T F	T F F
F	T	T	F T T	T T T	F T T
F	T	F	F T T	F T T	F T F
F	F	T	F T F	T F F	F T T
F	F	F	F T F	F T F	F T F

Invalid argument. It is because there is a case in which the premises are all true but the conclusion is false (row 2). Although there are other cases in which all premises and conclusion are true (rows 1, 5, 6, 8), since the argument does not guarantee that whenever all premises are true, the conclusion must be true, so the argument is invalid.

6 Let A = it is fair to smoke around non-smokers, B = second-hand cigarette smoke really is harmful, C = the Lung Association tells us that second-hand cigarette smoke is harmful

$B \rightarrow \neg A, B \vee \neg C \therefore C \rightarrow (\neg A \ \& \ B)$

A	B	C	B → ¬A	B ∨ ¬C	C → (¬A & B)
T	T	T	T F F	T T F	T F F F T
T	T	F	T F F	T T T	F T F F T
T	F	T	F T F	F F F	T F F F F
T	F	F	F T F	F T T	F T F F F
F	T	T	T T T	T T F	T T T T T
F	T	F	T T T	T T T	F T T T T

(*Continued*)

A	B	C	B → ¬ A	B ∨ ¬ C	C → (¬ A & B)
F	F	T	F T T	F F F	T F T F F
F	F	F	F T T	F T T	F T T F F

Valid, because there is no case in which the premises are true but the conclusion is false. In all cases in which both premises are true (rows 4, 5, 6, 8), the conclusion is also true.

7 Let A = she respects friends as individuals, B = she has many friends, C = she expects all her friends to behave alike.

B → A, A → ¬ C, B, ∴ ¬ C

A	B	C	B → A	A → ¬ C	B	¬ C
T	T	T	T	T F F	T	F
T	T	F	T	T T T	T	T
T	F	T	T	T F F	F	F
T	F	F	T	T T T	F	T
F	T	T	F	F T F	T	F
F	T	F	F	F T T	T	T
F	F	T	T	F T F	F	F
F	F	F	T	F T T	F	T

Whenever the premises are all true (row 2), the conclusion is also true. Therefore, it is a valid argument.

8 Let A = equality of opportunity is to be achieved, B = people previously disadvantaged should now be given special opportunities, C = some people receive preferential treatment

A → B, B → C, C → ¬ A, ∴ ¬ A

A	B	C	A → B	B → C	C → ¬ A	¬ A
T	T	T	T	T	T F F	F
T	T	F	T	F	F T F	F
T	F	T	F	T	T F F	F
T	F	F	F	T	F T F	F
F	T	T	T	T	T T T	T
F	T	F	T	F	F T T	T
F	F	T	T	T	T T T	T
F	F	F	T	T	F T T	T

Whenever the premises are all true (rows 5, 7, 8), the conclusion is also true. Therefore, the argument is **valid.**

9 Let A = people are entirely rational, B = all of a person's actions can be predicted in advance, C = the universe is essentially deterministic

A → (B ∨ C), ¬ B, ∴ ¬ C → ¬ A

A	B	C	A → (B ∨ C)	¬ B	¬ C → ¬ A
T	T	T	T T T	F	F T F
T	T	F	T T T	F	T F F
T	F	T	T T T	T	F T F
T	F	F	T F F	T	T F F
F	T	T	F T T	F	F T T
F	T	F	F T T	F	T T T
F	F	T	F T T	T	F T T
F	F	F	F T F	T	T T T

Whenever the premises are all true (rows 3, 7, 8), the conclusion is also true. Therefore, the argument is **valid.**

10 Let A = we learn, B = we know, C = we forget

A → B, C → B, C → ¬ A, ∴ ¬ A

A	B	C	A → B	C → B	C → ¬ A	¬ A
T	T	T	T	T	T F F	F
T	T	F	T	T	F T F	F
T	F	T	F	F	T F F	F
T	F	F	F	T	F T F	F
F	T	T	T	T	T T T	T
F	T	F	T	T	F T T	T
F	F	T	T	F	T T T	T
F	F	F	T	T	F T T	T

It is possible that all premises are true but the conclusion is false (row 2). Therefore, the argument is **invalid.**

Exercise 5.5: The short truth-table method

1 Assume the argument is invalid.

(A ∨ B) → C	(A & C) ≡ ¬B	B → ¬C
F T T T T	F F T T F T	T F F T

For the conclusion to be false, B is true and C is true. Put the values to other propositions and get A is false. No contradiction is found after all. The argument is invalid.

2 Assume the argument is invalid.

(A → B) & [(A & B) → C]	A → (C → B)	A → ¬B
T T T T T T T T T	T T T T T	T F F T

For the conclusion to be false, A is true and B is true. Put the values to other propositions and get C is true. No contradiction is found after all. The argument is invalid.

3 Assume the argument is invalid.

(A ∨ B) → C	D ∨ [C → (¬A & ¬B)]	A → (B ∨ D)
T TF T T	FT TT T$\underline{F}$TTF	$\underline{T}$F FFF

For the conclusion to be false, A is true, B is false and D is false. Put the values to the first premise, C is true. Put all values to the second premise, A has to be false. This contradicts the previous result.

Therefore, the assumption is mistaken and the argument is valid.

4 Assume the argument is invalid.

(A → B) & (C → ¬A)	A ∨ C	¬B ∨ A
F T T T T T T F	F T T	F T F F

For the conclusion to be false, B is true and A is false. Put the values to other propositions and get C is true. No contradiction is found after all. The argument is invalid.

5 Assume the argument is invalid.

A → (B ∨ C)	(¬ B → C) & A	C & ¬ B	A ≡ (B → C)
T T F T T	T F T T T T	$\underline{T}$T T $\underline{F}$	T F $\underline{T}$ F $\underline{F}$

For the third premise to be true, C is true and B is false. For the second premise to be true, A is true. Put all values to other propositions. For the conclusion to be false and A to be true, (B → C) has to be false and thus B is true and C is false. This contradicts previous results.

Therefore, the assumption is mistaken and the argument is valid.

Exercise 6.1: Writing ordinary sentences in predicate logic

1 Let S = being a boy scout, C = cheating
$\forall x\ (Sx \rightarrow \neg\ Cx)$

2 Let D = being a diplomat, R = being rich
$\exists x\ (Dx\ \&\ \neg\ Rx)$

3 Let S = being a spider bite, F = being fatal
$\exists x\ (Sx\ \&\ Fx)$

4 Let A = being an application, S = being successful
$\neg\forall x\ (Ax \rightarrow Sx)$, or equivalently, $\exists x\ (Ax\ \&\ \neg\ Sx)$

5 Let Q = being a qualified mechanic, R = can repair a car
$\forall x\ (Rx \rightarrow Qx)$, or equivalently, $\forall x\ (\neg\ Qx \rightarrow \neg\ Rx)$

6 Let S = can swim, D = can dive, R = can row a boat
$\forall x\ [Sx \vee \neg\ (Dx \vee Rx)]$, or equivalently $\forall x\ [Sx \vee (\neg\ Dx\ \&\ \neg\ Rx)]$, or $\forall x\ [\neg\ Sx \rightarrow \neg\ (Dx \vee Rx)]$, or $\forall x\ [(Dx \vee Rx) \rightarrow Sx]$

7 Let D = love detective stories, A = admire Sherlock Holmes, F = being a fool
$\forall x\ [Dx \rightarrow (Ax \vee Fx)]$

Admiration should normally be a two-place predicate Axy = x admires y. In this case the place of y is fixed to Sherlock Holmes and only the place of x is variable. Therefore, we may well treat this as a one-place predicate.

Another issue is whether the disjunction should be included as part of the consequent of the implication, or the main operation of the whole proposition is the disjunction (i.e. it connects the implication and an atomic proposition of 'one is a fool'). From understanding the meaning of the sentence, it seems that the 'one' in 'one is a fool' is linked to the person in 'anyone who loves detective stories' in the beginning. So the correct reading should be the former, not the latter.

8 Let b = Betty, $Lxyt$ = x loves y at time t
$\forall x \forall y \forall t \{Lbxt \rightarrow [(Lbxt \mathbin{\&} Lbyt) \rightarrow \mathrm{x} = \mathrm{y}]\}$

This is a complicated case because the proposition mentions loving a person at a certain time. So it means love here is a three-place predicate including time, rather than just a two-place predicate including two persons as we normally think of it.

The proposition mentions that Betty will love 'exactly one' person at a time. To express the meaning of 'exactly', it means if Betty loves someone else, that person would also be the same person Betty already loves. This is a discursive way to say that Betty will only love one person but not more.

Betty will 'only love' suggests that Betty may not love anyone at any given time, but if she loves someone, then she would only love one (so Betty will not be a two-timer). Based on this reading, we do not use existential quantification to indicate that there *is* someone Betty loves. Rather, we only use universal quantification: for any person, if Betty loves him/her, she will love only that person. We should also use universal quantification for time. It is because 'at a time' means 'at any time', rather than any particular point of time.

9 Let S = being a soldier, Oxy = x obeys y

$\forall x \exists y\ (Sx \rightarrow Oxy)\ /\ \therefore \exists y \forall x\ (Sx \rightarrow Oxy)$

In this case, the conclusion seems true because the institutional arrangement of the military makes it so. An army is highly bureaucratic and has a clear, single chain of command. Hence, it is true as a fact that, ultimately, a superior is supposed to be obeyed by every soldier. However, it is not

because the premise is true that the conclusion is true. Even though everyone obeys someone, the army can still be divided into factions or it is anarchy overall because there may not be one person recognized as the leader of everyone. So the argument, by the argument form alone, is not valid. The logical analogy mentioned in the text shows this already. To make this argument valid, some other premises are necessary.

10 Let S = being a soldier, $Uxy = x$ is a superior of y, $Oxy = x$ obeys y, $Pxy = x$ praises y

$\forall x \forall y\ [(Sx\ \&\ Uyx\ \&\ \neg\ Oxy) \rightarrow \neg\ Pyx]$

The proposition does not specify any particular superior a soldier disobeys. In fact, disobedience is a vice for a soldier and so any superiors may disapprove of and not praise a soldier who disobeys any of his superiors. Hence, we choose a universal quantifier for the superiors.

Questions 7–10 are more difficult. Do not worry if you cannot get the right answer right away. Just try to understand the analysis behind them and keep practising!

Exercise 6.2: Testing some simple inferences

1 $\forall x\ (Sx \rightarrow \neg\ Tx)$, $\exists x\ (Sx\ \&\ Ux)$ / $\therefore \exists x\ (Ux\ \&\ \neg\ Tx)$

1	$\forall x\ (Sx \rightarrow \neg\ Tx)$	
2	$\exists x\ (Sx\ \&\ Ux)$ / $\therefore \exists x\ (Ux\ \&\ \neg\ Tx)$	
3	$Sa\ \&\ Ua$	2 $\exists$E
4	Sa	3 Simp
5	Ua	3 Simp
6	$Sa \rightarrow \neg\ Ta$	1 $\forall$E
7	$\neg\ Ta$	6, 4 MP
8	$Ua\ \&\ \neg\ Ta$	5, 7 Conj
9	$\exists x\ (Ux\ \&\ \neg\ Tx)$	8 $\exists$I

2 $\exists x\,(Px \mathbin{\&} \neg Qx), \forall x\,(Px \to Rx) \mathbin{/} \therefore \exists x\,(Rx \mathbin{\&} \neg Qx)$

1	$\exists x\,(Px \mathbin{\&} \neg Qx)$	
2	$\forall x\,(Px \to Rx) \mathbin{/} \therefore \exists x\,(Rx \mathbin{\&} \neg Qx)$	
3	$Pa \mathbin{\&} \neg Qa$	1 ∃E
4	Pa	3 Simp
5	$\neg Qa$	3 Simp
6	$Pa \to Ra$	2 ∀E
7	Ra	6, 4 MP
8	$Ra \mathbin{\&} \neg Qa$	7, 5 Conj
9	$\exists x\,(Rx \mathbin{\&} \neg Qx)$	8 ∃I

3 $\forall x\,(Gx \to Fx), \forall x\,(Qx \to \neg Fx) \mathbin{/} \therefore \forall x\,(Qx \to \neg Gx)$

1	$\forall x\,(Gx \to Fx)$	
2	$\forall x\,(Qx \to \neg Fx) \mathbin{/} \therefore \forall x\,(Qx \to \neg Gx)$	
3	$Ga \to Fa$	1 ∀E
4	$\neg Ga \vee Fa$	3 Impl
5	$Fa \vee \neg Ga$	4 Com
6	$\neg Fa \to \neg Ga$	5 Impl
7	$Qa \to \neg Fa$	2 ∀E
8	$Qa \to \neg Ga$	7, 6 HS
9	$\forall x\,(Qx \to \neg Gx)$	8 ∀I

4 $\exists x\,\{(Ex \mathbin{\&} Fx) \mathbin{\&} [(Ex \vee Fx) \to (Gx \mathbin{\&} Hx)]\} \mathbin{/} \therefore \forall x(Ex \to Hx)$

1	$\exists x\,\{(Ex \mathbin{\&} Fx) \mathbin{\&} [(Ex \vee Fx) \to (Gx \mathbin{\&} Hx)]\} \mathbin{/} \therefore \forall x(Ex \to Hx)$	
2	$(Ea \mathbin{\&} Fa) \mathbin{\&} [(Ea \vee Fa) \to (Ga \mathbin{\&} Ha)]$	1 ∃E
3	$Ea \mathbin{\&} Fa$	2 Simp
4	$(Ea \vee Fa) \to (Ga \mathbin{\&} Ha)$	2 Simp

5	Ea	3 Simp
6	$Ea \vee Fa$	5 Add
7	$Ga \,\&\, Ha$	4, 6 MP
8	Ha	7 Simp
9	$Ha \vee \neg Ea$	8 Add
10	$\neg Ea \vee Ha$	9 Com
11	$Ea \rightarrow Ha$	10 Impl
12	$\forall x\,(Ex \rightarrow Hx)$	11 ∀I

5 $\exists x\,[(Cx \,\&\, \neg\,(Dx \rightarrow Ex))], \forall x[(Cx \,\&\, Dx) \rightarrow Fx], \forall\, x(Gx \rightarrow \neg\, Cx)\; /\; \therefore \exists x\,(\neg\, Gx \,\&\, Fx)$

1	$\exists x\,[(Cx \,\&\, \neg\,(Dx \rightarrow Ex))]$	
2	$\forall x\,[(Cx \,\&\, Dx) \rightarrow Fx]$	
3	$\forall x\,(Gx \rightarrow \neg\, Cx) / \therefore \exists x\,(\neg\, Gx \,\&\, Fx)$	
4	$Ca \,\&\, \neg\,(Da \rightarrow Ea)$	1 ∃E
5	Ca	4 Simp
6	$\neg\,(Da \rightarrow Ea)$	4 Simp
7	$\neg\,(\neg Da \vee Ea)$	6 Impl
8	$Da \,\&\, \neg\, Ea$	7 DeM
9	Da	8 Simp
10	$Ca \,\&\, Da$	5, 9 Conj
11	$(Ca \,\&\, Da) \rightarrow Fa$	2 ∀E
12	Fa	11, 10 MP
13	$Ga \rightarrow \neg\, Ca$	3 ∀E
14	$\neg\,\neg\, Ca$	5 DN
15	$\neg\, Ga$	13, 14 MT
16	$\neg\, Ga \,\&\, Fa$	15, 12 Conj
17	$\exists x\,(\neg\, Gx \,\&\, Fx)$	16 ∃I

6 Let P = makes the effort, R = receives appropriate rewards, a = Andrew

$\forall x\ (Px \rightarrow Rx), \neg\ Pa\ /\ \therefore \neg\ Ra$

Suppose there is only one individual in the domain. The argument is instantiated as follows.

$Pa \rightarrow Ra, \neg\ Pa\ /\ \therefore \neg\ Ra$

Assume this is an invalid argument,

$Pa \rightarrow Ra$	$\neg\ Pa$	$\neg\ Ra$
F T T	T F	F T

No contradiction is found. We found an instance in which the argument is invalid. Therefore, the quantified argument is indeed invalid.

7 Let M = being a manager, H = hardworking, O = being an officer

$\exists x\ (Mx\ \&\ Hx), \exists x\ (Ox\ \&\neg\ Hx)\ /\ \therefore \forall x\ (Ox \rightarrow \neg\ Mx)$

Try instantiation to one individual. Suppose a domain of only one individual a.

Assume the argument is invalid,

$Ma\ \&\ Ha$	$Oa\ \&\ \neg\ Ha$	$Oa \rightarrow \neg\ Ma$
T T $\underline{\text{T}}$	T T T $\underline{\text{F}}$	T F F T

There is a contradiction, so in this instance, the argument is valid on this instantiation.

Try instantiation to two individuals. Suppose a domain of only two individuals a and b.

Assume the argument is invalid,

$(Ma\ \&\ Ha) \vee (Mb\ \&\ Hb)$	$(Oa\ \&\ \neg\ Ha) \vee (Ob\ \&\ \neg\ Hb)$	$(Oa \rightarrow \neg\ Ma)\ \&\ (Ob \rightarrow \neg\ Mb)$
T T T T T F F	T F F T T T T T F	T F F T F T F F T

There is no contradiction. So the quantified argument is invalid.

8 Let V = being a vintage car, R = being rare, Oxy = x owns y
$\forall x\,(Vx \rightarrow Rx)\ /\ \therefore \exists x \exists y\,[(Oxy \,\&\, Vy) \rightarrow (Oxy \,\&\, Ry)]$

1	$\forall x\,(Vx \rightarrow Rx) / \therefore \exists x \exists y\,[(Oxy \,\&\, Vy) \rightarrow (Oxy \,\&\, Ry)]$	
2	$Vb \rightarrow Rb$	1 ∀E
3	$Oab \,\&\, Vb$	CP premise
4	Oab	3 Simp
5	Vb	3 Simp
6	Rb	2, 5 MP
7	$Oab \,\&\, Rb$	4, 6 Conj
8	$(Oab \,\&\, Vb) \rightarrow (Oab \,\&\, Rb)$	3-7 CP
9	$\exists x \exists y\,[(Oxy \,\&\, Vy) \rightarrow (Oxy \,\&\, Ry)]$	8 ∃I

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When you see black clouds looming and predict it's going to rain, you're using logic. When you decide that all swans are white because all the swans you've seen are white, that's logic too (even though the conclusion is false).

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